(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE BACHELOR OF SCIENCE

COURSE CODE: MAA 111/MAT 121

COURSE TITLE: **DIFFERENTIAL CALCULUS**

DATE: 21/07/2022 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

a) Given $g(x) = x^2 + 2x$, $f(x) =- x^2 + 4x$ and $h(x) = x - 2$
find $hogoh(x)$ (2mks)

b) Evaluate the following limits

i. $\lim\limits_{x\to 0} \frac{sin3x}{\sqrt{x+2}-\sqrt{2}}$ (3mks)

ii. $\lim\limits_{x\to 2} \frac{3x^2-x-10}{x^2-4}$ (3mks)

c) Evaluate

$$\lim_{x\to 9} \frac{x^2-81}{\sqrt{x}-3}$$ (3mks)

d) Prove that: $\lim\limits_{x\to 2} \frac{x^2-4}{x-2} = 4$ (3mks)

e) Determine if the following function is continuous at $x = 2$ (4 mks)

$$f(x) = \begin{cases} \frac{x^3-8}{x-2} & < 2 \\ 12 & x = 2 \\ x^3 + 4 & x > 2 \end{cases}$$

f) Find from the 1st principles or using the delta method the derivative of
$y = \sqrt{x}$ (4mks)

g) Find the equation of the tangent and normal to the curve $y = 2x^3 - 5x$ at the point $(-1,2)$
(5mks)

h) Differentiate $y = sinx^2$ (3mks)

QUESTION TWO (20 MARKS)

a) Prove the following theorems

i. $\lim\limits_{\alpha\to 0} \frac{sin\alpha}{\alpha} = 0$ (5mks)

ii. $\lim\limits_{\theta\to 0} \frac{cos\theta-1}{\theta} = 0$ (5mks)

b) Investigate the nature of the turning points of the curves
$y = 2x^3 - 3x^2 - 12x - 4$ (5mks)

QUESTION THREE (20 MARKS)

a) Find from the 1st principles or using the delta method the derivative of
$$y = 2x^3 + 2x^2 - x + 1$$ (6mks)

b) Find the slope of the line tangent to the graph of the equation
$x^3 + y^3 - 2xy^2 + yx + 2y = 1 + y^2$ at the point $(\frac{1}{2}, \frac{1}{3})$ (5mks)

c) Find y^I if $\sin(x + y) = y^2 cosx$ (4mks)

d) Find the Cartesian equation for each of the following parametric form.

$x = \frac{1}{1+t};\quad y = t^2 + 4$ (5mks)

QUESTION FOUR (20 MARKS)

a) Find the equation of the line tangent to the graph of $x(t) = t^2 + 1$ and $y(t) = \sqrt{1+t}$ at the pint $t = 3$ (10mks)

b) If $y = \frac{sinx}{x^2}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ hence prove that

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 + 2)y = 0$$ (10mks)

QUESTION FIVE (20 MARKS

a) The distance S moved by a body in t seconds is given by $S = 2t^3 - 13t^2 + 24t + 10$ find
   (i) The velocity when $t = 4$ seconds
   (ii) The value of t when the body comes to rest
   (iii) Find acceleration at 4 seconds (10 mks)

b) A particle P moves along a straight lineOX. At time $t = 0$ P is at the point O and t seconds later its displacement $S\,m$ is given by $S = 15t + 12t^2 - t^3$
   i. Write an expression for velocity and acceleration of P at t seconds.
   ii. Find when and where the particle is instantaneously at rest
   iii. Find when and where the particle will be when $\frac{d^2S}{dt^2} = 0$ (10mks)