(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS - 2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)

COURSE CODE: SPH 313

COURSE TITLE: MATHEMATICAL PHYSICS I

EXAM DURATION: 3 HOURS

DATE: 12/1/2022 **TIME:8-10AM**

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.
- Symbols have their usual meaning.

QUESTION ONE (30 MARKS)

(a) If $\boldsymbol{A} = 2i + j - 3k$ and $\boldsymbol{B} = 3i - 2j + 2k$ find

1) $(\mathbf{3A - 2B}) - \mathbf{2}(\mathbf{3B - 2A})$ **(2 marks)**
2) $(\mathbf{6A}).(\mathbf{B})$ **(3 marks)**
3) $\mathbf{3A \times 2B}$ **(3 marks)**

(b) What is the angle between $\hat{\imath} - 2k$ and $\hat{\imath} + 3\hat{\jmath}$ **(4 marks)**

(c) Given that $\boldsymbol{u} = 4x^2\boldsymbol{i} - y^3\boldsymbol{j} + z\boldsymbol{k}$, find

i. $\boldsymbol{\nabla}.\boldsymbol{u}$ **(2 marks)**
ii. $\boldsymbol{\nabla} \times \boldsymbol{u}$ **(3 marks)**

(d) Given that $\boldsymbol{A} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{B} = 2\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$, show that $\boldsymbol{A}.\boldsymbol{A} \times \boldsymbol{B} = 0$ **(3 marks)**

(e) If $\vec{A} = \hat{\imath} + 2\hat{\jmath} - \hat{k},\ \vec{B} = \hat{\jmath} + \hat{k}$ and $\vec{C} = \hat{\imath} - \hat{\jmath}$ show that $\vec{A} \times (\vec{B} \times \vec{C}) = -\vec{B} - \vec{C}$

(5 marks)

(f) Find the gradient of a potential V(r), if $\mathrm{V(r)} = \mathrm{V}(\sqrt{\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2}$ **(5 marks)**

QUESTION TWO (20 MARKS)

(a) A vector **r** is in the x-y Cartesian coordinate. If **r** has a fixed direction and the Cartesian coordinate is rotated in the counter-clockwise direction about z axis through angle q, such that we have $x' - y'$ coordinate axes. By using a diagram show that:

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$ **(4marks)**

Hence show that **r** is invariant under rotation **(3marks)**

(b) State Gauss' theorem **(2marks)**

(c) Prove Gauss' theorem stated in 2(b) above **(5marks)**

(d) One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that $[M_x, M_y] = iM_z$ **(6 marks)**

QUESTION THREE (20 MARKS)

(i) If $S(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$, find

(a) $\mathbf{\nabla S}$ at the point (1, 2, 3) **(4 marks)**

(b) The magnitude of the gradient of S, $|\mathbf{\nabla S}|$ at (1, 2, 3) **(2 marks)**

(ii) Show that, $\mathbf{\nabla}.\mathbf{\nabla} \times \mathbf{V} = 0$, if $V = V_x\hat{\imath} + V_y\hat{\jmath} + V_z\hat{k}$ **(4 marks)**

(iii) Show that the gradient of any scalar field $\phi(r)$ is Irrotational **(4 marks)**

(iv) One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that $M^2 \equiv M_x^2 + M_y^2 + M_z^2 = 2I$

Where I is a unit matrix **(6 marks)**

QUESTION FOUR (20 MARKS)

(i) Given that $x = \rho \cos \varphi$ and $y = \rho \sin \varphi$ for polar coordinate system. By using the Jacobian, find the area element of a polar coordinate system **(7marks)**

(ii) Given that $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$, and $z = r \cos \theta$ for spherical coordinate system. By using the Jacobian, find the area element of a spherical coordinate system **(7marks)**

(iii) By using the_Gauss elimination method, Solve $3x + 2y + z = 11, 2x + 3y + z = 13,$

$x + y + 4z = 12$ **(6marks)**

QUESTION FIVE (20 MARKS)

(i) A force is described by

$\mathbf{F} = -\boldsymbol{i} + \boldsymbol{j} - 6\boldsymbol{k}$

(a) Calculate the divergence of **F** **(4 marks)**

(b) calculate the curl of **F** **(6 marks)**

(ii) A particle moving in a circular orbit is given by a vector $\boldsymbol{r} = \boldsymbol{i} r \cos \omega t + \boldsymbol{j} r \sin \omega t$. Evaluate $\boldsymbol{r} \times \dot{\boldsymbol{r}}$, where r is the radius and ω is the angular velocity and both are constants **(4 marks)**

(iii) Prove that: $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$ **(6 marks)**