KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
SUPPLEMANTARY EXAMINATIONS

FOR THE DEGREE OF BSC(PHYSICS)

COURSE CODE: SPH 411

COURSE TITLE: COMPUTATIONAL TECHNIQUES IN PHYSICS

DATE: 12/1/2022 **TIME:** 11-1PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

Question One (30 marks)

(a) Define the inverse of a matrix A (1 mark)

(b) Find the inverse of the matrix $A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{pmatrix}$ (4 marks)

(c) Prove the properties stated below

$(A^{-1})^{-1} = A$ (1 mark)

$(A^{T})^{-1} = (A^{-1})^{T}$ (2 marks)

(d) Show that if $\lambda(x)$ is stationary then x is an eigenvector of A and $\lambda(x)$ is equal to the corresponding eigenvalue. (4 marks)

(e) Use the variation-of-parameters method to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. (4 marks)

(f) Find the characteristics of the one-dimensional wave equation $x^2 \frac{d^2u}{dx^2} - \frac{1}{c^2} x \frac{d^2u}{dt^2} = 0$. (4 marks)

(g) Show that $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$. (4 marks)

(h) Solve $2\frac{d^3y}{dx^3} + 6\frac{dy}{dx}\frac{d^2y}{dx^2} = x$. (4 marks)

(i) Find the complex conjugate of the matrix $B = \begin{pmatrix} 1 & 2 & 3i \\ 1+i & 1 & 0 \end{pmatrix}$. (2 marks)

Question Two (20 marks)

a) Find the eigenvalues and normalized eigenvectors of the real symmetric matrix

$$P = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix}$$ (11 marks)

b) Construct an orthonormal set of eigenvectors for the matrix

$$P = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$ (9 marks)

Question Three (20 marks)

Show that $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

(a) Using the relation $\left(\frac{\partial U}{\partial X}\right)_Y dX = \left(\frac{\partial U}{\partial Y}\right)_X dY$ (10 marks)

(b) Considering the potential $U - ST$. (10 marks)

Question Four (20 marks)

a) Using a unit circle and compute the value of π by

i) integration (10 marks)

ii) Monte Carlo Method with $N = 1000$ and $N_c = 7854$. (2 marks)

b) Given a one dimensional function of one variable, g : [a; b] → ℝ, Determine the Monte Carlo error in finding its integral. (8 marks).

Question Five (20 marks)

(a) State the general diffusion equation. (1mark)

(b) An infrared laser delivers a pulse of (heat) energy E to a point P on a large insulated sheet of thickness b, thermal conductivity k, specific heat s and density ρ. The sheet is initially at a uniform temperature. If $u(r,\ t)$ is the excess temperature a time t later, at a point that is a distance r (>> b) from P, then show that a suitable expression for u is $u(r,t) = \frac{\alpha}{t}\exp\left(\frac{r^2}{2\beta t}\right)$ where α and β are constants. Further,

(i) show that $\beta = 2k/(s\rho)$; (4 marks)

(ii) demonstrate that the excess heat energy in the sheet is independent of t, and hence evaluate α (3 marks)

(iii) prove that the total heat flow past any circle of radius r is E. (3 marks)

(c) Use LU decomposition to solve the set of simultaneous equations given below (9 marks)

$$2x_1 + 4x_2 + 3x_3 = 4$$

$$x_1 - 2x_2 - 2x_3 = 0$$

$$-3x_1 + 3x_2 + 2x_3 = -7$$