(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 424**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATIONS III**

DATE: 18/01/2022 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer **Questions ONE a**nd Any other **TWO**

QUESTION ONE [30MKS]

a. Give the definition of a fundamental matrix for the system of equation (2mks)

b. Find the limit cycle of the following systems and give their types (5mks)

   i. $r' = r(r-1)(r-2), \quad \theta' = 1$

   ii. $r' = r(r-1)^2, \quad \theta' = 1$

c. Find the general solution for the system of differential equations (5mks)

$$x' = x + y$$
$$y' = -2x + 4y$$

d. Define the flow of differential equations. (3mks)

e. Consider the system, $x' = -x + 2x^2 + y^2, \quad y' = -y + y^2$. Determine the basin of attraction (4mks)

f. Describe the stability of the zero solution of the system

$$\frac{dx}{dt} = x + 3y, \quad \frac{dy}{dt} = 2x$$

Using Lyapunov function (5mks)

g. Solve the initial value problem $x' = x, \quad x(0) = 1$ by Picard iteration (6mks)

QUESTION TWO [20MKS]

a. Let $A = \begin{pmatrix} 6 & 3 & -2 \\ -4 & -1 & 2 \\ 13 & 9 & -13 \end{pmatrix}$ (12mks)

   i. Evaluate e^{tA}

   ii. Solve the initial value problem $\vec{X}' = A\vec{x}, \quad \vec{x}(0) = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$

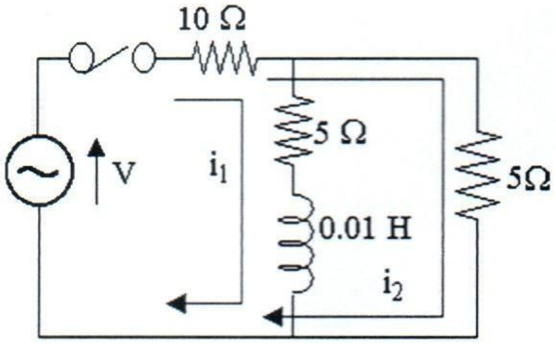
b. Assume that $X(x, t)$ is continuous and satisfies the Lipschitz condition on the interval $|t-a| \le T$ for all x, y. then the initial value problem $x' = X(x, t), \quad x(a) = c$ has a unique solution on the interval $|t-a| \le T$. Prove (8mks)

QUESTION THREE [20MKS]

a. Show that the solution of the system $\frac{dy}{dx} = y^2, \quad y(1) = -1$ can be extended (10mks)

b. In the two-mesh network shown below, the switch is closed at $t = 0$ and the voltage source is given by $V = 150 \sin 1000t$ V. Find the mesh currents i_1 and i_2 as given in the diagram.

(10mks)



QUESTION FOUR [20MKS]

a. Define the term exponential stability (3mks)

b. What are the conditions necessary for a path C that approaches the critical point $P(x_0,\ y_0)$ (2mks)

c. Investigate the stability of the zero solution of the system whose general solution is given by

$$x(t) = 3C_1 + C_2 e^{-t}$$
$$y(t) = 2C_1 t^2 e^{-t} - C_2 \cos t$$
(7mks)

d. Show that the point $(x_*,\ y_*)$ with coordinates $x_* = 2$ and $y_* = 5$ is the only equilibrium point and is a repeller for the following system of ODEs (8mks)

$$\begin{cases} x' = 10 - x - \dfrac{4xy}{1+x^2} \\ y' = x\left(1 - \dfrac{y}{1+x^2}\right) \end{cases}$$

QUESTION FIVE [20MKS]

a. Define the Lipchitz condition (2mks)

b. Find all the functions y_1 & y_2 such that (5mks)

$$y_1' = y_1 - 3y_2$$
$$y_2' = y_1 + 5y_2$$

c. Consider, $\dot{x} = x^2,\ \ x(0) = 1$. Show that the function $f(x) = x^2 \in C^1(\square)$ and the initial value *problem has a unique solution.* *(5mks)*

d. Consider the initial value problem $\dot{x} = t\,x,\ \ x(0) = 1$, study the continuity of the solution. (8mks)