(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OFBACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAA 224/MAT 321/MAA311**

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 13/01/2022   **TIME:** 8:00 – 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the terms;
    i.   Ordinary differential equation
    ii.   Partial differential equation   (4 marks)

b) State the order and degree of the following differential equations
    i. $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^{10} + 5y = 0$
    ii. $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = x^2 y$ (4 marks)

c) Show that $y = Cx^3$ is a solution of $x\frac{dy}{dt} - 3y = 0$ (5 marks)

d) Solve $\frac{dx}{dt} = e^t x$ give that $x(0) = 1$ (4 marks)

e) Prove that the following ordinary differential equations is homogeneous and find its solution.
$$xydx - (x^2 + y^2)dy = 0$$ (7 marks)

f) Solve the linear differential equation $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$ (6 marks)

QUESTION TWO (20 MARKS)

a) Solve the following ordinary differential equation using the method of integrating factor;
$$(x^2 + y^2 + x)dx + xydy = 0$$ (7 marks)

b) Test for exactness and solve the following ordinary differential equation
$$(xe^{xt} - 2x^3)dt + (te^{xt} - 6tx^2 - 2x)dx = 0$$ (7 marks)

c) Use the method of undetermined coefficient to solve
$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 2e^{3x}$$ (6 marks)

QUESTION THREE (20 MARKS)

a) Solve $(x^3 + 2y)dy + (3x^2y - 6x)dx = 0$ (6 marks)

b) Solve the following Cauchy-Euler equation
$$4x^2\frac{d^2y}{dx^2} + 12y = 0$$ (4 marks)

c) Use the method of variation of parameters to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$ (7 marks)

d) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Test for homogeneity and solve the following ordinary differential equation

$$(x - 2y + 1)dx + (4x - 3y - 6)dy = 0$$ (8 marks)

b) Solve $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$ (5 marks)

c) Solve the differential the following differential by first showing that it is of first degree $\frac{dy}{dx} = \frac{x+3y}{2x}$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Use the method of integrating factor to solve $\frac{dy}{dx} + \frac{2x+1}{x}y = e^{-2x}$

(5 marks)

b) Solve the Bernoulli's equation $\frac{dy}{dx} - \frac{1}{x}y = xy^2$ (6 marks)

c) A bacteria culture is known to grow at a rate proportional to the amount present. After 1 hour, 1000 strands of bacteria are observed in the culture, after 4 hours, 3000 strands are observed, find an expression for approximating number of strands of bacteria present in the culture at any time t (6 marks)

d) Find orthogonal trajectories to the circle $x^2 + y^2 = r^2$. (3 marks)

END