(38)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: STA 321/STA 342

COURSE TITLE: TEST OF HYPOTHESIS

DATE: 13/01/2022 **TIME:** 11:00 AM - 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1:

(a) Define a statistical Hypothesis (3 marks)

(b) Distinguish a simple from a composite Hypothesis (4 marks)

(c) State and explain two types of errors encountered in statistical testing of Hypotheses

(6 marks)

(d) Suppose that it is required that the mean operating life of size D batteries be 22 hours. Suppose also that the operating life of the batteries is normally distributed.It is known that the standard deviation of the operating life of all such batteries produced is 3 hours.If a sample of 9 batteries has a mean operating life of 20 hours, can we then conclude that the mean operating life of size D batteries is not 22 hours?Use the level of significance 5% to perform this test. (8 marks)

(e) Use the data shown in the following table below to test at 0.01 level of significance whether a person's ability in pure mathematics is independent of his or her interest in statistics

Ability in pure mathematics

Interest in Statistics		Low	Average	High
	Low	6	42	15
	Average	58	61	31
	High	14	47	29

(9 marks)

QUESTION 2: (20 marks)

(a) Let $\overline{X}$ and S^2 be the mean and the variance of a random sample of size n from a normal population with mean, μ and the variance, σ^2.
Prove that,

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has the t distribution with (n-1) degrees of freedom (10 marks)

(b) In 16 one-hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a standard deviation of 2.1 gallons. Test the claim that the average gasoline consumption of this engine is 12.00 gallons per hour. (7 marks)

(c) If S_1^2 and S_2^2 are the variances of independent random samples of size n_1 and n_2 from normal populations with variances σ_1^2 and σ_2^2, write an expression for the F distributed random variable, F. (3 marks)

QUESTION 3:

An experiment is performed to determine whether the average nicotine content of one kind of cigarette exceeds that of another kind by 0.20 milligram. If n1=50 cigarettes of the first kind had an average nicotine content of x1=2.61 milligrams with a standard deviation of s1=0.12 milligram, whereas n2=40 cigarettes of the other kind had an average nicotine content of x2=2.38 milligrams with a standard deviation of s2=0.14 milligram, test the null hypothesis $\mu 1-\mu 2=0.20$ against the alternative hypothesis $\mu 1-\mu 2\neq 0.20$ at 0.05 level of significance. Base the decision on the;

(i) P-value corresponding to the value of the appropriate test statistic

(ii) Critical values at the stated level of significance

(20 marks)

QUESTION 4:

(a) Sate three uses of the chi-square test. (3 marks)

(b) Suppose that the thickness of a part used in a semiconductor is its critical dimension and that measurements of the thickness of a random sample of 18 such parts have the variance of 0.68, where the measurements are in thousandths of an inch. The process is considered to under control if the variation of the thickness is given by a variance not greater than 0.36. Assuming that the measurements constitute a random sample from a normal population, test the null that the variance is equal to 0.36 against the alternative hypothesis that it is greater than 0.36, at 0.05 level of significance. Is the process under control? (8+2 marks)

(c) In comparing the variability of the tensile strength of two kinds of structural steel, an experiment yielded the following results; $n_1=13, S_1^2=19.2, n_2=16$ and $S_2^2=3.5$, where the measurements are in kilograms per square metre. Assuming the measurements constitute independent random samples from two normal populations, test

$H_0: \sigma_1^2=\sigma_2^2$ versus

$H_1: \sigma_1^2\neq\sigma_2^2$

at 0.02 level of significance (7 marks)

QUESTION 5:

(a) State and Prove the Neyman-Pearson's Lemma (10 marks)

(b) Examine whether a Best critical region (B.C.R) exists for testing $H_0: \vartheta=\vartheta_0$ against $H_1: \vartheta>\vartheta_0$ for the parameter ϑ of the distribution (10 marks)