KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS)

COURSE CODE: SPH 410

COURSE TITLE: MATHEMATICAL PHYSICS

DATE: 13/1/2022 **TIME: 11-1PM**

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

Question One

(a) State the associative and closure properties of the elements $\{X, Y, Z, ...\}$ belonging to a group G. (2 marks)

(b) Define the inverse X^{-1} of an element X belonging to a group G. (2 marks)

(c) Show that for the elements $\{X, Y, Z, ...\}$ belonging to a group G,

$$(X * Y * Z)(Z^{-1} * Y^{-1} * X^{-1}) = I. \qquad \text{(3 marks)}$$

(d) Given the rotation matrix $(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, show that $M(0) = I_2$. (3 marks)

(e) Show that the shortest curve joining two points is a straight line. (4 marks)

(f) Two rings, each of radius a, are placed parallel with their centres $2b$ apart and on a common normal. An open-ended axially symmetric soap film is formed between them. Find the shape assumed by the film. (4 marks)

(g) From Fermat's principle deduce Snell's law of refraction at an interface. (4 marks)

(h) Using Hamilton's principle derive the wave equation for small transverse oscillations of a taut string. (4 marks).

Question Two

(a) Define a group G. (10 marks)

(b) Given the expression $X^{-1} * (X * X^{-1})$, show that $X * X^{-1} = I$, where for a group G, $X \in G$, $X^{-1} \in G$ is the inverse of X and $I \in G$ is the identity element. (5 marks)

(c) A rotation matrix $M(\theta)$ is defined as $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ where $0 \leq \theta \leq 2\pi$. Show that $M(\theta)M(\varphi) = M(\theta + \varphi)$. (5 marks)

Question Three

(a) Define an Abelian group. (2 marks)

(b) Given the set $S = \{1,3,5,7\}$ under multiplication (mod 8):

i) Show that the set S forms a group. (10 marks)

ii) Generate a multiplication table for the group S. (2 marks)

(c) Define a homomorphism and state any two of its consequences. (6 marks)

Question Four

(a) Find the closed convex curve of length l that encloses the greatest possible area. (10 marks)

(b) A frictionless wire in a vertical plane connects two points A and B, A being higher than B. Let the position of A be fixed at the origin of an xy-coordinate system, but allow B to lie anywhere on the vertical line $x = x_0$. Find the shape of the wire such that a bead placed on it at A will slide under gravity to B in the shortest possible time. (10 marks)

Question Five

(a) A find the shape assumed by a uniform rope when suspended by its ends from two points at equal heights. (10 marks)

(b) Show that $\int_a^b y_j' p y_i' - y_j q y_i dx = \lambda_i \delta_{ij}$ (10 marks)