Completeness of Compact Operators Who's Norms Are Eigen Values

The class of compact operators is fundamental in operator theory. Characterization of compact operators acting on different spaces has been fascinating to many Mathematicians. In this paper we consider the Hilbert and Banach spaces. Let X be a Banach space and $T: X \to X$ be a linear operator, then T is compact if it maps bounded sequences in X to sequences with convergent subsequences. The eigenvalue of an operator T, is a scalar λ if there is a nontrivial solution x such that $Tx = \lambda x$. Such an x is called an eigenvector corresponding to the eigenvalue λ. The set of all eigenvalues of T is called the point spectrum of T. Abramorich, Aliprantis and Burkinshaw asserted that there is a sequence x_n of unit vectors such that $\lim_{n\to\infty}\left\| \|T\|x_n - Tx_n \right\| = 0$. A vector space V is said to be complete if every Cauchy sequence in Vconverges in V. In this paper, we have investigated in addition to completeness theorem other conditions for completeness of compact operators whose norms are eigenvalues. Let $\{T_n\}_{n\in\mathbb{N}}$ be an orthonormal sequence of compact operators whose norms are eigenvalues. Then it has been shown that the sequence $\{T_n\}_{n\in\mathbb{N}}$ is complete if and only if $\langle T, T_n\rangle = 0$ for all natural numbers $n$implies $T = 0$. Eigenvalues find a wide range of applications. In Biometrics, eigenfaces are used for identification. The norm is a notion of measurement, that is, the length or size of a mathematical object applicable in areas like statistics. Compact operators also have applications in Quantum mechanics.