(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 313**

COURSE TITLE: GROUP THEORY I

DATE: 25/05/2022 **TIME**: 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30 MARKS)

a) Define the following

   i. Proper Subgroup (2 marks)

   ii. Trivial subgroups (2 marks)

   iii. Normal subgroup (1 marks)

   iv. Factor group (3 marks)

b) State the conditions under which a subset H of a group G can be a subgroup (3marks)

c) Let G be a group and $a, b \in G$. Show that the equation $ax = b$ has a unique solution(5mks)

d) Let G be a group. Show that $x * z = y * z \Rightarrow x = y$ for $x, y \in G$ (5marks)

e) Let H be a subgroup of a group G. Show that the left cosets of H in G Partition G.(6mks)

f) Let H be the subgroup of Z_6 consisting of the elements 0 and 3. determine the cosets of H in G. (3marks)

QUESTION TWO (20 MARKS)

a) Define the following

   i. Permutation (1 marks)

   ii. Symmetric group (2marks)

   iii. Alternating group (2 marks)

b) Show that every permutation can be expressed as a product of transpositions. (3marks)

c) Compose the following permutations in cycle notation: (1234)*(13)(24) (3 marks)

d) Show that every permutation can be expressed as a product of transpositions (3marks)

e) Let K be the subgroup of S3 defined by the permutations {(1), (12)}. Find the left and right cosets (6marks)

QUESTION THREE (20 MARKS)

a) Define the following

   i. Transposition (1 marks)

   ii. Odd permutation (1marks)

   iii. Simple group (2marks)

   iv. Composition series (3marks)

b) Show that every cyclic group is abelian (5 marks)

c) Show that every subgroup of a cyclic group is cyclic (8 marks)

QUESTION FOUR (20MARKS)

a) Define the following

i. Center of a group (2 marks)

ii. Homomorphism (2marks)

iii. Automorphism (2 marks)

b) Show that the center Z of the group G is a normal subgroup of G (5marks)

c) If $\phi: G \rightarrow H$ is Homomorphism, then $\text{Im}(\phi) \cong G/\ker(\phi)$

i. Show that i is well defined (5marks)

ii. Show that i is a homomorphism (4marks)

QUESTION FIVE (20 MARKS)

a) Define the following

i. Conjugacy class (2marks)

ii. Centralizer (2marks)

iii. Faithful action (1marks)

b) Show that the orbits of an action partition X. (4 marks)

c) Show that if $|G| = n$, then there is an embedding $G \hookrightarrow S_n$. (5marks)

d) Show that stab(x) is a subgroup of G for each $x \in X$ (3 marks)

e) Show that $|Gx| = (G:\text{Stab}(x))$ (3marks)