KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE IN BSC (PHYSICS)

COURSE CODE: SPC 411

COURSE TITLE: QUANTUM MECHANICS II

DATE: 20/05/2022 **TIME: 2:00PM-4:00PM**

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE [30 MARKS]

a) Show that $\sigma_x\sigma_y = i\sigma_z$ where $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ are Pauli's spin matrices. [5 marks]

b) Find the Eigen function of the operator:-$\hat{L}_z = -i\hbar\frac{d}{d\varphi}$. [5 marks]

c) Use variational method to find the ground state energy of the one dimensional harmonic oscillator $H = \frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2$ using the trial wave function $\psi(x) = Ae^{-bx^2}$ where b is a constant and A is determined by normalization. [5 marks]

d) The amplitude of scattering by spherically symmetric potential $V(r)$ with momentum transfer q is given by:-$A = \int_0^\infty \frac{sin(qr/\hbar)}{qr/\hbar}V(r)4\pi r^2 dr$. Use Born approximation to show that for the case of Yukawa-type potential, this leads to an amplitude proportional to $(q^2 + m^2c^2)^{-1}$. [5 marks]

e) Show that:- $[\hat{L}_x, \hat{p}_y]\psi = i\hbar p_z\psi$ [5 marks]

f) Calculate the scattering angle in laboratory frame of reference between two photons if it is 10^0 in the centre of mass frame. [5 marks]

QUESTION TWO [20 MARKS]

a) State the approaches used in the theory of scattering. [2 marks]

b) Give any four particles which may be used in scattering experiments. [2 marks]

c) What information regarding particles may be gathered in 2(b) above? [4 marks]

d) Define the terms:- incident flux and impact parameter. [2 marks]

e) Use the WKB method to estimate the energy of a one dimensional harmonic oscillator. [10 marks]

QUESTION THREE [20 MARKS]

Show that in spherical coordinates the angular momentum operator is given by: - $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$.

QUESTION FOUR [20 MARKS]

a) An electron is in spinor state $\chi = A\begin{bmatrix} 3i \\ 4 \end{bmatrix}$ hence:-

i) Determine the normalization constant A. [2 marks]

ii) Find the expectation values $\langle S_x\rangle$, $\langle S_y\rangle$ and $\langle S_z\rangle$. [6 marks]

iii) Find the uncertainties $\sigma_{S_x}^2$, $\sigma_{S_y}^2$ and $\sigma_{S_z}^2$. [6 marks]

b) Use first order perturbation theory to calculate the energy of the nth excited state for a spinless particle of mass m moving in an infinite well of length 2L with walls at x = 0 and x = 2L where:- [6 marks]

$$V(x) = \begin{cases} 0 & 0 \le x \le 2L \\ \infty & otherwise \end{cases}, V_p(x) = \lambda V_0\delta(x - L) \text{ and } \lambda \ll 1$$

QUESTION FIVE [20 MARKS]

a) Find the ground state energy and wave function of a system of N-interacting identical particles that are confined to a one dimensional [12 marks] infinite well when the particles are bosons and spin $\frac{1}{2}$ fermions.

b) Consider a system of three non-interacting identical spin $\frac{1}{2}$ particles that are in the same spin state $\left|\frac{1}{2},\frac{1}{2}\right\rangle$ and confined to move in a one-dimensional infinite potential well of length a and potential $V(x) = \begin{cases} 0 \; for \; 0 < x < a \\ \infty \quad otherwise \end{cases}$ find the energy and wave function of the ground state and first excited state. [8 marks]