(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAA 121/MAT 102**

COURSE TITLE: FOUNDATION MATHEMATICS II

DATE: 16/05/22 **TIME:** 2.00 PM -4.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) With the aid of examples define; (6 mks)

(i) A vector

(ii) A square matrix

(iii) A row matrix

(b) Given that $A = \begin{bmatrix} -2 & 6 & -6 \\ 3 & 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 22 \\ -5 & 3 & -4 \end{bmatrix}$ find $(-AB^T)^T$ (4 mks)

(c) If $\boldsymbol{a} =- 2i - 3k$ and $\boldsymbol{b} = 4i - j + 2k$ evaluate $2\boldsymbol{b}.(\boldsymbol{a} \times \boldsymbol{b})$ (5 mks)

(d) Find A if $= \begin{bmatrix} -5 & 7 \\ -3 & 0 \end{bmatrix} = (A^T + 2I)^{-1}$ (5 mks)

(e) Find the solution of the following system of linear equations using augmented matrices (10 mks)

$$x_1 + x_2 + x_3 =- 2$$
$$2x_1 - 2x_2 + 3x_3 - 3x_4 =- 20$$
$$x_1 - x_2 + 2x_3 - x_4 =- 8$$
$$x_1 - x_2 + 4x_3 + 3x_4 = 4$$

QUESTION TWO (20 MARKS)

(a) Find β for which $\boldsymbol{p} = \beta i + \beta j - 3k$ and $\boldsymbol{p} = \beta i + 5j + 2k$ are perpendicular (3 mks)

(b) Show that $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|sin\theta$ with $0 \leq \theta \leq \pi$ (5 mks)

(c) Given that $M = \begin{bmatrix} -2 & 6 \\ 3 & -3 \end{bmatrix}$ and $N = \begin{bmatrix} -10 & 2 \\ 7 & -1 \end{bmatrix}$

Prove that $\det(MN) = \det M \det N$ (4 mks)

(d) Reduce the system into row-echelon form hence by backward substitution solve it (8 mks)

$$x - y + z = 5$$
$$7x + 5y - z = 8$$
$$y + 2x + z = 7$$

UESTION THREE (20 MARKS)

(a) Given $\boldsymbol{a} = \langle 2, -1, -2 \rangle$ and $\boldsymbol{b} = \langle -6,0,1 \rangle$ compute

(i) $\boldsymbol{a} \times \boldsymbol{b}$ (2 mks)

(ii) $(\boldsymbol{b} \times -3\boldsymbol{a}).\boldsymbol{b}$ (5 mks)

(b) If $detA = 8$ and $\det B = -3$ calculate $det(B^{-1}A^TB^2A^3)$ given that A and B are square matrices (5 mks)

(c) Determine if the two vectors are parallel, orthogonal or neither $2i - j + 3k$ and $2i - 3j - 5k$ (3 mks)

(d) Compute the rank of $\begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{bmatrix}$ (6 mks)

QUESTION FOUR (20 MARKS)

(a) Given $A = \begin{bmatrix} 3\alpha & 11\alpha \\ -1 & \alpha \end{bmatrix}$ has determinant of -10 find α (3 mks)

(b) Compute the determinant of $\begin{bmatrix} 6 & -3 & -5 & 3 \\ -5 & 1 & 0 & 1 \\ 3 & -1 & 2 & 2 \\ -1 & -3 & 3 & -1 \end{bmatrix}$ (6 mks)

(c) Use Cramer's rule to find x_1, x_2, and x_3, (10 mks)

$$4x_1 + 2x_2 + 2x_3 = 0$$

$$2x_1 + 6x_2 + 2x_3 = 1$$

$$2x_1 + 2x_2 + 5x_3 = 0$$

QUESTION FIVE (20 MARKS)

(a) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ (10 mks)

(b) Given that

$$B = \begin{bmatrix} -6 & -4 & 3 \\ 1 & 0 & -5 \\ -1 & 3 & 2 \end{bmatrix}$$ Compute $B(adjB)$ (10 mks)

END