

256



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
FORTH YEAR FIRST SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAA 412

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 24/05/2022

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Form a partial differential equation associated with the primitive $z = f(x^2 + y^2)$.
(4 marks)
- b) Solve the Lagrange's Linear Equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (7 marks)
- c) By direct integration, solve $\frac{\partial^2 z}{\partial y^2} = z$; $y = 0$ then $z = e^x$, $\frac{\partial z}{\partial y} = e^{-x}$ (6 marks)
- d) By eliminating arbitrary functions, obtain the partial differential equation from:
 $z = f(x + ct) + g(x - ct)$. (8 marks)
- e) Solve the non-linear partial differential equation of the form $x^2 p^2 + y^2 q^2 = z^2$. (5 mks)

QUESTION TWO [20 MARKS]

- (a) A Lagrange's Linear Partial differential equation is of the form
 $Pp + Qq = R$, where P, Q and R are functions of x, y, z and $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. Show that
its solution is given by $f(u, v) = 0$. (12 marks)

- (b) Using the method of Multipliers, find the complete solution of the differential equation
 $x(z^2 - y^2) \frac{\partial z}{\partial x} + y(x^2 - z^2) \frac{\partial z}{\partial y} = z(y^2 - x^2)$ (8 marks)

QUESTION THREE [20 MARKS]

- (a) Find a complete integral of the partial differential equation $p(q^2 + 1) + (b - z)q = 0$
by Charpit's method. (12 marks)
- (b) By eliminating arbitrary functions, obtain the partial differential equation from:
 $z = f(x + ct) + g(x - ct)$. (8 marks)

QUESTION FOUR [20 MARKS]

- (a) Classify the partial differential equation $\frac{\partial^2 z}{\partial x^2} + x \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} = 0$. (6 marks)
- (b) Using the method of separation of variables, solve;
 $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, when $u(x, 0) = 6e^{-3x}$. (14 marks)

QUESTION FIVE [20 MARKS]

- (a) Obtain solution for the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = P_0 \cos pt$, (P_0 is a constant) when $x = l$ and $y = 0$ when $x = 0$. (14 marks)
- (b) Solve $(w + y + z) \frac{\partial w}{\partial x} + (w + x + z) \frac{\partial w}{\partial y} + (w + x + y) \frac{\partial w}{\partial z} = x + y + z$ (6 marks)