



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
3RD YEAR FIRST SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAP 314

COURSE TITLE: NUMBER THEORY

DATE: 19/05/2022

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

Question 1: 30 marks

- a) If $n \in \mathbb{Z}$, prove that $0 \cdot n = 0$ (5 marks)
- b) There are infinitely many primes. Prove (4 marks)
- c) Show that if $a \equiv b \pmod{n}$ then $b = a + nq$ for some integer q , and conversely. (4 marks)
- d) Prove that $2^0 + 2^1 + 2^2 + \dots + 2^{n-1} = 2^n - 1$. (8 marks)
- e) Solve $6x + 9y = 21$. (4 marks)
- f) Find all the prime numbers up to 100 using sieve of Erastosthens. (5 marks)

Question 2: 20 marks

- a) Prove that $\sqrt[3]{2}$ is not a rational number. (8 marks)
- b) Prove that if x is even, then $x^2 + 2x + 4$ is divisible by 4. (5 marks)
- c) Prove that if $n \in \mathbb{Z}$, then n^2 does not have a remainder of 2 or 3 when it is divided by 5. (7 marks)

Question 3: 20 marks

- a) The greatest common divisor of any two numbers a and b , which are not simultaneously zero, exists and is unique. It is the biggest among the common divisors of a and b . Prove. (5 marks)
- b) Show that an integer p is prime iff it is irreducible. (4 marks)
- c) Let p be a prime number and suppose that p divides ab where a and b are integers. Then either p divides a or p divides b (or both). Prove. (11 marks)

Question 4: 20 marks

- a) Find $17^{341} \pmod{5}$. (5 marks)
- b) Solve for x : $5x \equiv 1 \pmod{12}$ (5 marks)
- c) Prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$ for every positive integer n . (10 marks)

Question 5: 20 marks

- a) Simon buys large shirts for \$ 18 each and small shirts for \$ 11 each. The shirts cost a total of \$ 1188. What is the smallest total number of shirts he could have bought? (10 marks)
- b) Let p be prime, and suppose p does not divide a . then $a^{p-1} \equiv 1 \pmod{p}$. (10 marks)