180

(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **STA 311/STA 341**

COURSE TITLE: THEORY OF ESTIMATION

DATE: 23/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (16MKS)

a) Give definitions of the following terms as used in theory of estimation

i) Parametric space (1mk)

ii) Action space (1mk)

iii) Loss function (1mk)

b) For a binomial distribution with p.d.f $f(x,p) = \begin{cases} p^x(1-p)^{1-x} \\ 0 \ elsewhere \end{cases} \quad x = 0,1$

Show that, T= $\sum x_i$ is complete (3mks)

c) Let $f(x;\mu) = \begin{cases} \frac{1}{2\pi} e^{-\frac{1}{2}(x-\mu)^2} \\ 0 \ elsewhere \end{cases}$

Obtain the estimator of μ which attains C.R lower bound (3mks)

d) Consider a random sample of size n ,$x_1, x_2, x_3, \ldots . x_n$ drawn from a population with mean μ and variance σ^2

$$\textit{Let } T_1 = \frac{1}{3}(x_1 + x_2 + x_3) \quad \textit{and} \quad T_2 = \frac{1}{2}(x_4 + x_5)$$

i) Show that $T_1$and T_2 are unbiased estimators for μ (2mks)

ii) Show that T_1 is more efficient estimator for μ than T_2 (3mks)

e) Show that a sample mean $\bar{x}$ of a random sample of size n ,

having a p.d.f $f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & , 0 < x < \infty \\ o \ elsewhwere, & 0 < \theta < \infty \end{cases}$

Is an unbiased estimator for θ (4mks)

f) Consider a random sample from a poison distribution given by,

$f(x,\lambda) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{x=1,2.......} \\ 0 \ elsewhere \end{cases}$

Find the maximum likelihood estimate for λ (4mks)

g) A random sample $x_1, x_2, \ldots, x_n$ of size 5 is drawn from a normal distribution with known mean. Consider the following estimators to estimate

i) $T_1 = \frac{x_1+x_2+x_3+x_4+x_5}{5}$

ii) $T_2 = \frac{x_1+x_2}{2} + x_3$

iii) $T_3 = \frac{x_1+x_2+\lambda x_3}{3}$

a) Find λ (2mks)

b) Are T_1 and T_2 unbiased (3mks)

c) State giving reasons which estimate is the best (3mks)

QUESTION TWO: (20MKS)

a) Name two methods of finding estimators (2mks)

b) Let L (B_0, B_1) = s $=\sum [y_i - E(y_i)]^2$.

Where $E(y_i) = B_0 + B_1 x$

i) Find the least squires estimates of B_0 *and* B_1 (7mks)

ii) Show that $\widehat{B}_1$ *and* $\widehat{B}_1$ are unbiased estimates of B_0 *and* B_1 If $\widehat{B}_0 = \overline{Y} - \widehat{B}_1\overline{X}$ and $B_1 = \frac{\sum x_i(y_i - \bar{y})}{\sum x_i(x_i - \bar{x})}$ Assuming x is a non-random variable (6mks)

iii) Find the variance of $\widehat{B}_0$ from b above (2mks)

iv) Let $\hat{\theta} = \frac{\sum y_i(x_i - \bar{x})}{\sum (x_i - \bar{x})}$ find the variance of the estimate of θ (3mks)

QUESTION THREE (20 mks)

a) Define

i) sufficient statistics T (2mks)

ii) complete statistics T (2mks)

b) Let $x_1, x_2, \ldots x_n$ be a random sample from N (μ, σ^2), $-\infty < x < \infty$.

Find the MVUE for μ where σ^2 is known. (4mks)

c) assuming that

$$\tilde{\theta}_1 = \tilde{\theta}_2 = a_1 y_1 + a_2 y_2 + a_3 y_3$$

and E $(Y_1) = \theta_1$, E$(Y_2) = \theta_1 + \theta_2$ and E$(Y_3) = \theta_1 + 2\theta_2$

i) Find $\tilde{\theta}_1$ and $\tilde{\theta}_2$ using least square estimators. (8mks)

ii) Find the mean of $\tilde{\theta}_1$ and $\tilde{\theta}_2$ (2mks)

iii) Find the variance of $\tilde{\theta}_1$ and $\tilde{\theta}_2$ (2mks)

QUESTION FOUR (13mks)

a) Let X be a random variable with the density function f(x;θ), θ is unknown parameter.

Define

i) An un biased estimator of $\boldsymbol{\theta}$. (3marks)

ii) statistics T (3marks)

b) if E$(T_1) = \boldsymbol{\theta}$ and E$(T_2) = \boldsymbol{\theta}$, V$(T_1) = \sigma_1^2$ and , V$(T_2) = \sigma_2^2$ where T_1 and T_2 are independent statistics, show that T= $\boldsymbol{\lambda} T_1 + (1- \boldsymbol{\lambda})T_2$ is unbiased estimator of θ for which the variance is minimized when

$$\boldsymbol{\lambda} = \frac{\delta_2^2}{\delta_1^2 + \delta_2^2}$$ (8mks)

c) Let $X_1, X_2, ..., X_n$ be random sample from population f(x; $\boldsymbol{\theta}$), $\boldsymbol{\theta}$ is an unknown parameter. if δ(x) =($X_1, X_2, ..., X_n$) is a decision or action taken for the parameter $\boldsymbol{\theta}$, then define

i) Loss function (2 mks)

ii) Risk function (2 mks)

iii) Point estimator (2 mks)

QUESTION FIVE (20MKS)

a) State and proof crammer- Rao inequality (7mks)

b) Let $X_{1,}$ $X_{2,...,}X_n$ be a random sample f(x;**λ**) = $\frac{e^{-\lambda}\lambda^x}{x!}$, for x=0,1,2,... taking Ψ(**λ**) = $e^{-\lambda}$ find Crammer – Rao lower bound for T, where T is an unbiased for Ψ(**λ**) (4mks)

c. Let $X_{1,}$ $X_{2,...,}X_n$ be a random sample from a population with density f(x; μ, δ^2), where (μ, δ^2), are unknown parameters. Find jointly the sufficient statistics for (μ, δ^2). (4mks)

d) Let x be a Bernoulli random variable with parameter p, p is unknown. Show that T= $\sum x_i$ is complete statistics for p. (5mks)