(Knowledge for Development)

KIBABII UNIVERSITY

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UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 242/STA 221

COURSE TITLE: PROBABILITY AND DISTRIBUTION MODELS

DATE: 09/05/2022 **TIME:** 9:00 aM - 11:00 aM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE: (30 MKS)

a) Let x be a random variable with density function given by;

$$f(x)=\begin{cases} \lambda e^{-\lambda x} & x>0 \\ 0 \ otherwise \end{cases}$$

i) Find the moment generating function of x (3mks)

ii) the mean of x (2mks)

iii) variance of x (3mks)

b) Two random variables x and y have the following j.p.d.f; determine if the random variables are independent. (8mks)

$$f(x,y)=\begin{cases} 2-x-y & 0<x<1, \ 0<y<1 \\ 0 \ elsewhere \end{cases}$$

c) Let $f(x)=\begin{cases} 1 & 0<x<1, \\ 0 \ elsewhere \end{cases}$

If $y=3x$,

i) Find the Jacobean of the transformation (2mks)

ii) obtain the distribution of y (4mks)

d) A fair coin and die were tossed and rolled respectively, let the event that a head was obtained be given by x and the event that a 6 was obtained be given by y, are the two events independent? (8mks)

QUESTION TWO: (20MKS)

Let x and y be continuous random variables with joint probability distribution function given by,

$$f(x,y)=\begin{cases} \frac{2}{3}(x+2y) & 0<x<1, 0<y<1 \\ 0 \ elsewhere \end{cases}$$

Find;

a) $E(x)$ (2mks)

b) $E(y)$ (2mks)

c) $E(x^2)$ (2mks)

d) $E(y^2)$ (2mks)

e) $E(xy)$ (3mks)

f) $cov(xy)$ (2mks)

g) $var(x)$ (2mks)

h) $var(y)$ (2mks)

i) Correlation coefficient of x and y (2mks)

j) Are the random variables independent? (1mk)

QUESTION THREE: (20 MKS)

Let $y_1, y_2, y_3, \ldots, y_n$ be in a real product space; consider y_1, y_2 *and* y_3 with joint probability distribution function given by;

$$f(y_1, y_2, y_3) = \begin{cases} \frac{1}{(2\pi)^{\frac{3}{2}}} e^{-\frac{1}{2}(y_1^2+y_2^2+y_3^2)} & -\infty < y_1 < \infty, \\ & -\infty < y_2 < \infty \\ & -\infty < y_3 < \infty \\ 0 \; else \; where & \end{cases}$$

Find the;

a) Marginal density function of

i) $f(y_1)$ (2mks)

ii) $f(y_2)$ (2mks)

iii) $f(y_3)$ (2mks)

b) Conditional density function of

i) $f(y_1/y_2, y_3)$ (2mks)

ii) $f(y_2/y_1, y_3)$ (2mks)

iii) $f(y_3/y_1, y_2)$ (2mks)

i) $E(y_1/y_2, y_3)$ (2mks)

ii) $E(y_2/y_1, y_3)$ (2mks)

iii) $E(y_3/y_1, y_2)$ (2mks)

c) Determine if y_1, y_2, y_3 are stochastically independent. (2mks)

QUESTION FOUR: (20MKS)

Let x and y have the following joint distribution

X ↓	Y →	-3	2	4	Totals
1		0.1	0.2	0.2	0.5
3		0.3	0.1	0.1	0.5
Total		0.4	0.3	0.3	1

Find;

i) The distribution of x and y (4mks)

ii) The covariance of x and y (8mks)

iii) The correlation coefficient of x and y (6mks)

iv) Are the random variables independent? explain (2mks)

QUESTION FIVE: (20MKS)

(a) Find the moment generating function a gamma density function defined as

$$f(x) = \frac{\lambda^{\eta}}{\Gamma(\eta)} x^{\eta-1} e^{-\lambda x}; \quad x \geq 0$$ (6mks)

(b) Using the moment generating function obtained in (a) above and assuming $\eta = 1$ *and* λ *as* 0.025 find the mean and standard deviation of density function (8mks)

(c) Show that

$$m_x(t) = \frac{1}{(1 - \beta t)^{\alpha}}$$

for a continuous density function

$$f(x) = \frac{1}{\beta\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}; \quad x \geq 0$$

where $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx$ (6mks)