(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(RENEWABLE ENERGY)

COURSE CODE: MAP 351

COURSE TITLE: ENGINEERING MATHEMATICS III

DATE: 25/05/2022 **TIME:** 9:00 AM -11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30marks)

(a). Define the following terms

(i). Laplace transform of a function $f(t)$ **(1 Mark)**

(ii). Dirac delta a function **(1 Mark)**

(iii). Conservative force field **(2 Mark)**

b). Find the inverse Laplace transform of $\frac{s+3}{(s+1)(s+2)}$ **(4 Marks)**

c). (i). Show that the following limit does not exists

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2$$ **(4 Marks)**

(ii). Given that $f(x, y) = y\cos(xy)$ find f_{xy} and f_{yy}. **(4 Marks)**

d). Find the Fourier cosine series for $f(x) = L - x$ on $0 \le x \le L$ **(5 Marks)**

e). Find the Taylor 4th polynomial for $f(x) = e^{-x}$ at $x = 0$. **(5 Marks)**

f). Define a Heaviside function, hence find its Laplace transform. **(4 Marks)**

QUESTION TWO (20 MKS)

a). Differentiate between odd and even functions giving an example in each case **(4 Marks)**

b). Using green's theorem evaluate $\oint_C (y - \sin x)dx + \cos x \ dx$ where C is the triangle whole vertices are $O(0,0)$, $A\left(\frac{\pi}{2}, 0\right)$ and $B\left(\frac{\pi}{2}, 1\right)$. **(6 Marks)**

c). Find the Fourier series for $f(x) = \begin{cases} L, \ if \ -L \le x < 0 \\ 2x, if \ \ 0 \le x \le L \end{cases}$ on $-L \le x \le L$. **(10 Marks)**

QUESTION THREE (20 MKS)

a). Find the Laplace transform of $f(t) = \cos at$ **(6 Marks)**

b). Find the Fourier Sine series of $f(x) = x$ on $-L \le x \le L$. **(4 Marks)**

c). Find the $\boldsymbol{curl}\ (\boldsymbol{curl}\ \boldsymbol{v})$ given that $\boldsymbol{v} = x^2y\boldsymbol{i} - 2xz\boldsymbol{j} + 2yz\boldsymbol{k}$. **(4 Marks)**

d). Using stokes theorem evaluate $\iint_S (\nabla \times \boldsymbol{A}) d\boldsymbol{S}$ where $\boldsymbol{A} = (x + y)\boldsymbol{i} + (2y - x)\boldsymbol{j} + z^2\boldsymbol{k}$ and S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$. **(6 Marks)**

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$	e^{-cs} / [illegible]	26.	$\delta(t-c)$	e^{-cs}