



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
FOURTH YEAR FIRST SEMESTER
SPECIAL/SUPPLIMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION
AND BACHELOR OF SCIENCE

COURSE CODE: MAT 423

COURSE TITLE: ODE II

DATE: 12/01/2022

TIME: 8:00 AM -10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE 30 Marks

- a) Classify the differential by degree, linearity, initial value problem or Boundary value problem
- (i) $x \frac{d^2 y}{dx^2} + 5 \left(\frac{dy}{dx} \right)^2 + 8y^3 = e^x, y(0) = y'(0) = 0$ (2 marks)
- (ii) $\frac{d^2 y}{dx^2} = xy, y(0) = 0, y^1(1) = 2$ (2 marks)
- b) (i) State the existence and uniqueness theorem for nth order differential equation. (3 marks)
- (ii) Prove that there exists a unique solution for the equation:
 $3x \frac{d^2 y}{dx^2} + 5x^2 \frac{dy}{dx} - 8y = e^x, y(2) = 2, y'(2) = 3.$ (3 marks)
- c) Solve the non linear differential equation: $xy'' = 4x - 2y'$ (4marks)
- d) Find a power series solution of differential equation $(x - 2)y' + y = 0$ (5 marks)
- e) Given the differential equation: $(x - y)dx - xdy + zdz = 0,$
- (i) Show that it is integrable (3 marks)
- (ii) Solve the equation (4 marks)
- f) Find the Legendre's polynomial of order three. (4 marks)

QUESTION TWO 20 Marks

- a) Given the differential equation: $(x + 1)^2 \frac{d^2 y}{dx^2} - 3(x + 1) \frac{dy}{dx} + 3y = 0$
- (i) Prove that $y = x + 1$ is a solution. (1 marks)
- (ii) Find the general solution by reducing the order. (7 marks)
- b) Given the differential equation $4xy'' + 8y' + xy = 0$. Find
- (i) The exponents of the differential equation. (5 marks)
- (ii) Prove that $y = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx$ is equal to $y = d_1 e^{(a+ib)x} + d_2 e^{(a-ib)x}$ using Euler's formula (5marks)
- (iii) The Frobenius solution of the differential equation. (2 marks)

QUESTION THREE 20 Marks

- a) Given the differential equation: $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0.$
- (i) Show that e^{2x} and e^{3x} are linearly independent of this equation. (2 marks)
- (ii) Find the solution that satisfies the conditions $y(0) = 2, y'(0) = 3$ (2 marks)
- b) State Rodrigue's formula for the nth order Legendre's polynomial. Find $P_2(x)$ and $P_3(x)$ using Rodrigue's formula. (5 marks)

- c) Given the differential equation: $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$
- (i) Show that it is integrable (4 marks)
- (ii) Solve the equation (7 marks)

QUESTION FOUR 20 Marks

- a) Classify the point $x=0$ as ordinary, regular singular or an irregular singular point for the following equations;
- (i) $3x^3y'' + 2x^2y' + (1 - x^2) = 0$ (2 marks)
- (ii) $xy'' + x^2y' + (e^x - 1) = 0$ (2 marks)
- b) Find the power series solution of the differential equation: $y'' + y = 0$ (8 marks)
- c) Solve the differential equation: $yy'' = (y')^2$ (8 marks)

QUESTION FIVE 20 Marks

- a) Given the differential equation $2x^2y'' + 3xy' - y = 0$ with $y(1) = y'(1) = 1$.
- (i) Prove that the equation has a unique solution. (3 marks)
- (ii) Show that $y = \frac{1}{x}$ is a solution of the differential equation. (2 marks)
- b) Find the general solution of the differential equation $(x^2 - 1)y'' - 2xy' + 2y = 0$ given that $y = x$ is a solution. (8 marks)
- c) Solve the nonlinear differential equation: $x^2y \frac{d^2y}{dx^2} + \left(x \frac{dy}{dx} - y\right)^2 - 3y^2 = 0$. (7 marks)