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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAA 225

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 19/05/2022

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define and give an example of following types of singularities
- (i) Connectedness (2 mks)
 - (ii) Neighborhood (2 mks)
 - (iii) Limit point (2 mks)
 - (iv) Point set (2 mks)
 - (v) Complex function (2 mks)
- (b) Convert the following into polar co-ordinates
- (i) $z = 3 + 2i$ (3 mks)
 - (ii) $z = -2 + 3i$ (3 mks)
- (c)(i) State De-moivres theorem and use it to prove that
 $\cos 3\alpha = \cos^3 \alpha - 3\cos \alpha \sin^2 \alpha$ and $\sin 3\alpha = 3\cos^2 \alpha - \sin^3 \alpha$ (6 mks).
- (ii) Solve the equation $x^4 = 1$. (8 mks)

QUESTION TWO (20 MARKS)

- (i) Show that for the complex variable z , the following formula is valid ;
 $\cos^2 z + \sin^2 z = 1$ (8 mks)
- (ii) Prove that $\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$ (8 mks)
- (iii) Using the definition of Limit ,show that ;
 $\lim_{z \rightarrow i} (7z - 1) = 7i - 1$ (4 mks)

QUESTION THREE (20 MARKS)

- (a) (i) Let $W = f(z) = z^2$, find the value of W which corresponds to $z = -2 + i$ and show the correspondence can be represented graphically . (5 mks)
- (ii) Prove that $1 - \tanh^2 z = \operatorname{sech}^2 z$ (5 mks)
- (b) Show whether the following functions are analytic or not in the entire complex plane ;

(i) $W = e^z$ (5 mks)

(ii) $f(z) = x^2 - y^2 + i(3x^2y - y^3)$ (5 mks)

QUESTION FOUR (20 MARKS)

(a) Evaluate $\int_{0,3}^{2,4} (2y + x^2)dx + (3x - y)dy$ along

(i) The parabola $x = 2t$, $y = t^2 + 3$ (6 mks)

(ii) Straight lines from (0,3) to (2,3) and then from (2,3) to (2,4) . (5 mks)

(iii) Straight line from (0,3) to (2,4) . (5 mks)

(b) Define the following terms ;

(i) Simply connected (2 mks)

(ii) Multiply connected (2 mks)

QUESTION FIVE (20 MARKS)

(a) Evaluate $\oint_C (5x + 6y - 3)dx + (3x - 4y + 2)dy$ around a triangle in the xy plane with vertices at (0,0) and (4,3). (10 mks)

(b) Prove that the series $z(1 - z) + z^2(1 - z) + z^3(1 - z) + \dots$ converges for $|z| < 1$ and find its sum. (10 mks)