(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 212/MAP 222**

COURSE TITLE: **REAL ANALYSIS I**

DATE: 10/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms:

(i)	Open neighbourhood	(2 marks)
(ii)	Countable set	(2 marks)
(iii)	Inverse of a function	(2 marks)
(iv)	Bounded set	(2 marks)
(v)	Rational function	(2 marks)

b) Show that the empty set $\emptyset$ is always open. (5 marks)

c) Give 3 conditions that need to be satisfied for a function f(x) to be continous at a point. (3 marks)

d) Show that if a, b $\in \mathbb{R}$ such that $a \leq b + \varepsilon$ for every $\varepsilon > 0$, then $a \leq b$ (3 marks)

e) If $x \in \mathbb{R}$ and $x \neq 0$. Show that $x^{-1} > 0$ iff $x > 0$. (5 marks)

f) Show that if T is anon empty set of areal number with Sup say b, then for all element $a < b$ $\exists$ $x \in T$ such that $a < x \leq b$ (5 marks)

QUESTION TWO (20 MARKS)

a) Define the following terms:

(i)	Open set	(3 marks)
(ii)	Interior point	(5 marks)

b) Show that if $\{E_\alpha : \alpha \in \Lambda\}$ is any family of closed subsets of X with respect to (X, f), then $\bigcap E_\alpha$ is closed in (X, f). (7 marks)

c) Show that $\emptyset$, X are always closed in (X, f). (5 marks)

QUESTION THREE (20 MARKS)

Let A, B, C be subsets of a universal set U. Show that:

(i) A-(B ∪ C) = (A-B) ∩ (A-C) (6 marks)

(ii) If A = B then (A ⊆ B) ∧ (B ⊆ A) (4 marks)

(iii) $(A \cup B)^c = A^c \cap B^c$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Show that if x and y are positive real numbers (x, y ∈ ℝ) then x < y iff $x^2 < y^2$ (8 marks)

b) Show that if (a, b ∈ ℝ) then | a | ≤ b iff –b < a < b (6 marks)

c) Show that if (a, b ∈ ℝ) then | a + b | ≤ | a | +| b | (6 marks)

QUESTION FIVE (20 MARKS)

a) Let A and B be non-empty sets of real numbers. Let C be the set such that

$c = \{x + y : x \in A \text{ and } y \in B\}$.

Show that if A and B have supremums then C also has a supremum and

Sup C = Sup A + Sup B (10 marks)

b) Considering the function f(1, -∞)→(0, 1) defined by f(x) = $\frac{x-1}{x+1}$, show that f posses an inverse $f^{-1} = \frac{y+1}{y-1}$, (6 marks)

c) Let f: ℝ → ℝ defined by f(x) = $\frac{x^2-1}{x^2+1}$ and g: ℝ → ℝ defined by g(x) = x^3. Find g. f (x) (4 marks)