



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021/2022 ACADEMIC YEAR**  
**FORTH YEAR FIRST SEMESTER**  
**MAIN EXAMINATION**  
**FOR DEGREE OF BACHELOR OF**  
**SCIENCE MATHEMATICS**

**COURSE CODE: MAP 412**

**COURSE TITLE: MEASURE THEORY**

**DATE: 20/05/2022**

**TIME: 9:00 AM - 11:00 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer question ONE and any other TWO questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**QUESTION ONE (20 MARKS)**

- a) Define the following terms
- i. Ring
  - ii. Algebra
  - iii.  $\sigma$ -ring
- b) Given that  $R$  is a ring of a set  $Y$ . show that the monotone class of  $R$ ,  $M(R)$  coincides with the ring generated by  $R$ ,  $G(R)$  ie  $M(R) = G(R)$
- c)
- i. Show that if  $\mu_F$  is the contradiction of a measure,  $\mu$  by a fixed set  $F$  in a ring  $R$ , then  $\mu_F$  is a measure in  $R$
  - ii. Show also if  $\mu_F < \infty$ , then  $\mu_F$  is a finite measure

**QUESTION TWO (20 MARKS)**

- a) Define the following terms
- i. Hereditary
  - ii. Hereditary  $\sigma$ -ring
  - iii. Outer measure
  - iv.  $\nu$ -measure
- b) Show that if  $\nu$  is an outer measure, then the class  $M$  of  $\nu$ -measurable sets is a ring
- b) Show that if  $\mu$  is measurable on a ring  $R$ , and  $M$  is the class of all  $M^*$ -measurable sets, then  $G(R) \subset M$  and the restriction of  $M^*$  to  $G(R)$  is a measure  $\bar{\mu}$  extending  $\mu$

**QUESTION THREE (20 MARKS)**

- a) Define the following terms
- i. Lebesgue measurable
  - ii. Lebesgue measure
- d) Show that if  $\alpha_n \uparrow \alpha$  and  $\beta_n \uparrow \beta$  then  $\alpha_n + \beta_n \uparrow \alpha + \beta$
- b) Show that if  $(\mu_j)$  is an increasingly directed family of measures on a ring  $R$ , and  $\mu$  is the set function on  $R$  defined by the formula  $\mu(E) = LUB \mu_j(E)$  then  $\mu$  is a measure on  $R$

**QUESTION FOUR (20 MARKS)**

- a) Show that if  $f_n$  is a sequence of integrable functions such that  $f_n \geq 0$  a.e and  $\liminf \int f_n du < \infty$  then there exists an integrable function  $f$  such that  $f = \liminf f_n$  a.e and one has  $\int f du \leq \liminf \int f_n du$
- b) Suppose  $|f_n| \leq |f|$  a.e ( $n = 1, 2, \dots$ ) where the  $f_n$  and  $g$  are integrable functions then there exists an integrable function  $f$  such that  $f = \liminf f_n$  a.e. moreover,  $|f| \leq |g|$  a.e and  $\int f du \leq \liminf \int f_n du$

**QUESTION FIVE (20 MARKS)**

- a) Show that if  $\alpha_n \uparrow \alpha$  and  $\beta_n \uparrow \beta$  then  $\alpha_n \beta_n \uparrow \alpha \beta$
- b) Show that if  $f, g \in L^2$  then  $|(f|g)| \leq \|f\|_2 \|g\|_2$  (Cauchy -schwarz inequality)
- c) Show that if  $f, g \in L^2$  then  $\|f + g\|_2 \leq \|f\|_2 + \|g\|_2$  (Triangle inequality)