(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: **STA 411**

COURSE TITLE: TIME SERIES ANALYSIS

DATE: 23/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

*This Paper Consists of **4 Printed Pages**. Please Turn Over.*

QUESTION 1: (30 Marks)

a) Explain the following terms as used in time series analysis:

i) Stationary process [1mk]
ii) Stationarity in the weak sense [1mk]
iii) Moving average process [1mk]
iv) Autoregressive process [1mk]
v) White noise process [1mk]

b) Find the auto covariance function ($\boldsymbol{\sigma(h)}$) and the autocorrelation function ($\boldsymbol{\rho(h)}$) of a moving average process of order q (MA(q)). [8mks]

c) Consider autoregressive process of order 1 (AR (1)) given by
$X_t = \alpha X_{t-1} + e_t$, where $\propto$ is a constant.

i) If $|\alpha| < 1$, show that X_t may be expressed as infinite order of a MA process. [4mks]

ii) Find its auto-covariance function $(\sigma(h))$ and its autocorrelation function $(\rho(h))$. [3mks]

d) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where
$Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$ and $X_t = e^{i\lambda t}$
and state the changes in its amplitude, wavelength and phase angle. [5mks]

e) Find the spectral density function of an AR (1) process given by
$X_t = \alpha X_{t-1} + e_t$, where $|\alpha| < 1$ [5mks]

QUESTION 2: (20 Marks)

a) Suppose we have data up to time $n(x_1, x_2, \dots, x_n)$

i) Show that minimum mean squared error forecast of x_{n+k} is the conditional mean of x_{n+k} at time n.
i.e. $\hat{x}(n, k) = E(x_{n+k}/x_1, x_2, \dots, x_n)$ [6mks]

ii) Consider the AR(1) model $X_t = \propto X_{t-1} + e_t$, $|\propto| < 1$.
Forecast x_{n+3}. [2mks]

b) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that
$X_t = e^{i\lambda t}$ and $Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$
Where

$$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \mp 1, \mp 2, \dots\dots\dots, \mp m \\ 0, & otherwise \end{cases}$$

[12mks]

QUESTION 3: (20 Marks)

a) Consider an AR(1) process with mean μ given by
$X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \dots\dots$

Find the estimates of the parameters α and μ using the method of least squares. [8mks]

b) Consider a second order process AR (2) given by
$X_t = {}^1/_3 X_{t-1} + {}^2/_9 X_{t-2} + e_t$.
Show that this process is stationary and hence obtain its ACF [12mks]

QUESTION 4: (20 Marks)

a) i) Briefly describe the main objectives in the analysis of a time series. [3mks]

ii) State the unique feature that distinguishes time series from other branches of statistics. [1mk]

iii) Identify the main stages in setting up a Box-Jenkins forecasting model. [4mks]

b) Show that the AR(2) process given $\boldsymbol{X_t = X_{t-1} - {}^1/_2 X_{t-2} + e_t}$ is stationary and hence find its ACF. [12mks]

QUESTION 5: (20 Marks)

a) If an observed values $(X_1, X_2, ..., X_n)$ on a discrete time series forms $n-1$ pairs of observation $(X_1, X_2), (X_2, X_3), ..., (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable

Find:

i) The correlation coefficient r_1 between X_t and X_{t-1} [5mks]

ii) The correlation between observations at a distance k apart. [2mks]

b) Consider an AR(2) process given by $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + e_t$.

Write down the Yule-Walker equations and hence, find the first two values of autocorrelation functions $\rho(1)$ and $\rho(2)$ if $\alpha_1 = 0.75$ and $\alpha_2 =- 0.25$ [6mks]

c) Consider a moving average process given by $\boldsymbol{X_t = e_t + \beta e_{t-1}}$, where $(\boldsymbol{\beta_0 = 1, \beta_1 = 1})$.

Find its spectral density function. [7mks]