(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAP 211/MAT 201/MAT 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 11/02/2021 **TIME: 8 AM- 10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Consider the system of equations

$a_1x_1 + b_1x_2 = c_1$
$a_2x_1 + b_2x_2 = c_2$
$a_3x_1 + b_3x_2 = c_3$

Discuss the relative positions of the above three lines when

(i) the system has no solutions, (2 marks)
(ii) the system has exactly one solution, (2 marks)
(iii) the system has infinitely many solutions. (2 marks)

b) Solve the following linear system using elementary row operations on the augmented Matrix (6 marks)

$x_1 - 2x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$-4x_1 + 5x_2 + 9x_3 = -9$

c) Let $T: IR^2 \to IR^3$ be given by $T(x,y)=(x,y,1)$. Show that T is not linear (3 marks)

d) Show that if u is a non-zero vector then the length of the vector $\frac{\mathbf{u}}{||\mathbf{u}||}$ is 1 (5 marks)

a) Given that $\mathbf{u} = (1,1,3)$ and $\mathbf{w} = (3,1,2)$, find

i) $\mathbf{u}_1$, the projection of **u** onto **w** (5 marks)
ii) $\mathbf{u}_2$, the perpendicular vector to **w** (3 marks)

b) Find the standard matrix of $T: IR^3 \to IR^2$ defined by (2 marks)

$$T\begin{matrix} x \\ y \\ z \end{matrix} = \begin{pmatrix} x-2y+z \\ x-y \end{pmatrix}$$

QUESTION TWO (20 MARKS)

a) Given a vector $\mathbf{v} = (a, b, c,)$ in IR^3

i) Show that $\cos \alpha = \frac{a}{||\mathbf{v}||}$ (2 marks)

ii) Find $\cos \beta$ (2 marks)

iii) Find $\cos \gamma$ (2 marks)

iv) Show that $\frac{\mathbf{v}}{\|\mathbf{v}\|} = (\cos \alpha, \cos \beta, \cos \gamma)$ (2 marks)

v) Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ (2 marks)

b) Let V be a vector space, $\mathbf{u} \epsilon V$ and α is a scalar. Prove that the following properties hold.

i) $0\mathbf{u} = 0$ (2 marks)

ii) $\alpha 0 = 0$ (2 marks)

iii) $(-1)\mathbf{u} = -\mathbf{u}$ (2 marks)

iv) If $\alpha\mathbf{u} = 0$ then $\alpha = 0$ or $\mathbf{u} = 0$ (4 marks)

QUESTION THREE (20 MARKS)

a) Let $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (2,-3, 1)$ and $\mathbf{w} = (3, 2,-1)$

i) Find the components of the vector $\mathbf{u}-3\mathbf{u}+8\mathbf{w}$ (2 marks)

ii) Find the scalars c_1, c_2, c_3 such that $c_1\mathbf{u}+c_2\mathbf{v}+c_3\mathbf{w} = (6, 14,-2)$ (6 marks)

b) Let $\mathbf{u} = (2,-1, 1)$, $\mathbf{v} = (1, 1, 2)$. Find $\langle \mathbf{u}, \mathbf{v} \rangle$ and the angle between these two vectors. (3 marks)

QUESTION FOUR (20 MARKS)

c) Given that $\mathbf{u} = (2,-1,3)$ and $\mathbf{w} = (4,-1,2)$, find

iii) $\mathbf{u}_1$, the projection of $\mathbf{u}$ onto $\mathbf{w}$ (5 marks)

iv) $\mathbf{u}_2$, the perpendicular vector to $\mathbf{w}$ (3 marks)

d) Given that $\mathbf{u} = (2,-1, 1)$ and $\mathbf{v} = (1, 1,-1)$, show that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. (2 marks)

e) If $\mathbf{u} = (1,2,-2)$ and $\mathbf{v} = (3,0,1)$ find the cross product $\mathbf{u} \times \mathbf{v}$ (5 marks)

f) Let $\mathbf{u} = (1, 2,-2)$ and $\mathbf{v} = (3, 0, 1)$. Show that $\langle \mathbf{u}, \mathbf{u} \times \mathbf{v} \rangle$ and $\langle \mathbf{v}, \mathbf{u} \times \mathbf{v} \rangle = 0$ and hence $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$. (5 marks)