(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 322**

COURSE TITLE: **GROUP THEORY II**

DATE: 01/10/21 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

a. Define the following
   i. Center of a Group (2marks)
   ii. Conjugacy Classes (2marks)
   iii. P-groups (2marks)
   iv. Sylow p-subgroup (2marks)
   v. Normalizer (2marks)

b. State the following
   i. Class Equation (2 marks)
   ii. Cauchy Theorem (2 marks)
   iii. First Sylow Theorem (2 marks)
   iv. Second Sylow Theorem (2 marks)

c. Determine the Conjugacy classes and the class equation in S_3 (5 marks)

d. Let G be a group of order p^n where p is prime. Show that G has a non-trivial center (7 marks)

QUESTION TWO (20MARKS)

a. Define the following sets
   i. Maximal normal subgroup (2marks)
   ii. Composition series (2marks)
   iii. Soluble group (2marks)

b. Show that every finite group G has a composition series (7marks)

c. Show that all finite abelian groups are soluble (7marks)

QUESTION THREE (20MARKS)

a. *State the following theorems*
   i. Nilpotency class (2marks)
   ii. Central series (2marks)
   iii. Lower central series (2marks)

b. Show that every nilpotent group is solvable (4marks)

c. Show that a group G is nilpotent if and only if it has a central series (4marks)

d. If G is a finite group and P is a Sylow p-subgroup of G. Show that $N_G(N_G(P)) = N_G(P)$ (6marks)

QUESTION FOUR (20MARKS)

a. State the following theorems
   i. Fundamental Theorem of Finite Abelian Groups (3marks)
   ii. The Fundamental Theorem of Finitely Generated Abelian Groups (3marks)

b. Let H be the subgroup of a group G that is generated by $\{g_i \in G: I \in I\}$. Show that $h \in H$ exactly when it is a product of the form $h = g_{i1}^{\alpha 1} \ldots\ldots g_{in}^{\alpha n}$ where the $g_{ik}s$ are not necessarily distinct (8marks)

c. Classify all abelian groups of order $540 = 2^2.3^3.5$ using the fundamental theorem of finite abelian groups (6marks)

QUESTION FIVE (20MARKS)

a. Define the following
   i. External direct product (2marks)
   ii. Internal direct product (2marks)
   iii. Internal semi direct product (2marks)

b. Show that if G is the internal direct product of H and K, then G is isomorphic to the external direct product H×K (9marks)

c. Let G be a group with subgroups H and K. Suppose that G = HK and H∩K = {1}. Show that every element g of G can be written uniquely in the form hk for h∈H and k∈K (5marks)