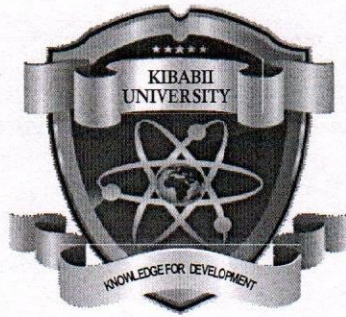


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(Knowledge for Development)

KIBABII UNIVERSITY

(KIBU)

**UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR**

**SPECIAL/SUPPLEMENTARY EXAMINATIONS
YEAR ONE SEMESTER ONE EXAMINATIONS**

**FOR THE DEGREE OF
BACHELOR OF SCIENCE
(COMPUTER SCIENCE)**

COURSE CODE : CSC 112

**COURSE TITLE : DISCRETE
STRUCTURES I**

DATE: 12/09/2017

TIME: 08:00 A.M – 10:00 A.M

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTIONS ONE AND ANY OTHER TWO

QUESTION 1 (COMPULSORY)**[30 MARKS]**

- a) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then prove that $a+c \equiv b+d \pmod{m}$ [3 marks]
- b) How many ways can a committee of 3 faculty members and 2 students be selected from 7 faculty members and 8 students? Show your work. [4 marks]
- c) Two fair six-sided dice are rolled and the sum s of the numbers coming up is recorded. What is the probability that $s \geq 10$? Show your work. [3 marks]
- d) Given some predicate $P(x_1, x_2, \dots, x_n)$ concerning some universe U , explain the following concepts. [3 marks]
- $P(x_1, x_2, \dots, x_n)$ is valid in U
 - $P(x_1, x_2, \dots, x_n)$ is satisfiable in U
 - $P(x_1, x_2, \dots, x_n)$ is unsatisfiable in U
- e) What are the two ways we can bind variables in a predicate? [2 marks]
- f) Given that the universe of discourse is the set of Integers, write the following in English and state its truth value. [2 marks]

$$\exists y \forall x [x + y = 0]$$

- g) Show that $(p \rightarrow q) \rightarrow r$ and $(p \rightarrow (q \rightarrow r))$ are not equivalent. [4 marks]
- h) Consider the following recurrence relation and the initial condition:
 $F(n) = F(n-1) + 2$ for $n \geq 1$.
 $F(n) = 0$ for $n = 0$
- List the first 5 elements of the sequence defined by this recurrence relation. [2 marks]
 - Solve this recurrence relation, i.e. find the general formula for $F(n)$ [2 marks]
- i) Briefly define the following terms. [2 marks]
- An injective (one to one) function.
 - An onto function
- j) Let $A = B = \{a; b; c\}$. Consider the relation $g = \{(a; b); (b; c); (c; c)\}$. [3 marks]
 Is g one-to-one? Is g onto? Why?

QUESTION 2**(20 marks)**

- a) Given the Universe as the set of Integers and,
 $N(x)$: "x is a nonnegative integer"
 $E(x)$: "x is even"
 $O(x)$: "x is odd"
 $P(x)$: "x is prime number"
 Write the following in symbolic logic notation:
- Every integer is even or odd [1 mark]
 - All prime integers are nonnegative [2 marks]
 - The only even prime is two [2 marks]
 - Not all integers are odd [1 mark]
 - Not all primes are odd [2 marks]
 - If an integer is not odd, then it's even [2 marks]
- b) A certain island is populated by only two types of people: morans, who always tell the truth, and morons, who always lie. Two of the inhabitants of the island tell you the following:

A says: Both of us are morons
B says nothing.

What are A and B? (Note: make sure that you show that your answer is the only consistent solution.) [5 marks]

- c) Assume the two statements “Oil company Tullow does not make huge profits if the price of a barrel of oil does not exceed \$60 per barrel” and “Oil company Tullow makes huge profits” are both true. What, if anything, can you conclude about the price of a barrel of oil? Show how you arrive at the conclusion. [5 marks]

QUESTION 3 [20 marks]

- a) Define the following types of proof: [5 marks]
- Vacuous proof
 - Trivial proof
 - Direct proof
 - Indirect proof
 - Proof by contradiction
- b) A perfect number is number whose sum of its factors, excluding itself, is the number itself e.g. 6, 28 etc. Using indirect proof, prove that a perfect number is not prime. [3 marks]
- c) Using proof by contradiction, prove that $\sqrt{2}$ is not a rational number. [12 marks]

Question 4 [20 marks]

- a) Let $A \times B = \{(1, 1), (2, 2), (3, 1), (3, 2), (1, 2), (1, 4), (2, 1), (2, 4), (3, 4)\}$. Find the power set of B, $P(B)$. [5 marks]
- b) Suppose A is the set of distinct letters in the word *elephant*, B is the set of distinct letters in the word *sycophant*, C is the set of distinct letters in the word *fantastic*, and D is the set of distinct letters in the word *student*. The universe U is the set of 26 lower-case letters of the English alphabet. Find [12 marks]
- $A \cup B$
 - $A \cap C$
 - $A \cap (C \cup D)$
 - $(A \cup B \cup C \cup D)'$
 - Find two finite sets A and B such that $A \in B$ and $A \subset B$.
- c) Use a Venn diagram to show that the set equation $P \cap (Q \cup R) = (P \cap Q) \cup R$ is not valid. [3 marks]

Question 5 [20 marks]

- a) Define the expression $a \equiv b \pmod{m}$ [2 marks]
- b) Compute $6^{16} \pmod{5}$ [3 marks]
- c) Find all integer solutions for the linear congruence. [8 marks]
- $$8x \equiv 7 \pmod{13}$$
- d) Use the Euclidean algorithm to show that $\gcd(19, 141) = 1$ [7 marks]