(Knowledge for Development)

KIBABII UNIVERSITY
(KIBU)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS

END OF SEMESTER EXAMINATION

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELORS OF SCIENCE IN

(COMPUTER SCIENCE)

COURSE CODE: CSC 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 12/05/2022 TIME: 2.00 P.M. – 4.00 P.M. 2HRS

INSTRUCTIONS TO CANDIDATES:

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

Paper Consists of 5 Printed Pages. Please Turn Over

QUESTION ONE (COMPULSORY) [30 MARKS]

a. Briefly explain the difference between computation and deduction and explain the connection of the two to logic. **[5 marks]**

b. What is symbolic logic? Give the general pattern used in representing symbolic logic. **[3 marks]**

c. Differentiate between Modus Ponens and Modus Tollens using relevant arguments. **[4 marks]**

d. Use the Euclidean algorithm to find the greatest common divisor of 46 and 21.hence or otherwise find integers s and t satisfying that gcd(46, 21) = s(46) + t (21). **[4 marks]**

e. Determine all integers x such that $x \equiv 2 \pmod{46}$ and $x \equiv 1 \pmod{21}$. **[4 marks]**

f. The number of bacteria, double every hour, then what will be the population of the bacteria after 10 hours? **[2 marks]**

g. Suppose E is an event in a sample space S with P (E) > 0. Define probability that an event A occurs once E has occurred or the conditional probability of A given E. **[2 marks]**

h. Let A and B be mutually exclusive events. Define both product and sum rule of A and B. **[2 marks]**

i. A pair of fair dice is thrown. Find the probability that the sum is 10 or greater if:
5 appears on the first dice; **[2 marks]**
5 appears on at least one dice. **[2 marks]**

QUESTION TWO [20 MARKS]

a. Three students Andrew, Brian and Christian are accused of introducing a virus in the SCI computer Lab. During the interrogation they make the following claims:

Andrew says: "*Brian did it and Christian is innocent*"
Brian says: "*If Andrew is guilty then Christian is guilty too*".
Christian says: "*I did not do it. One of the others or maybe both of them did it*"

i. Write a formula in propositional logic then represents the conjunction of the three above claims using the following atomic propositions: A: Andrew is guilty, B: Brian is guilty and C: Christian is guilty. **[3 marks]**

ii. Are the three above statements contradictory? Justify. **[3 marks]**

iii. Assuming that nobody lied, who is innocent and who is guilty? Justify **[2 marks]**

b. Differentiate between propositional and predicate logic and explain any two limitations of propositional logic that can be overcome by predicate logic. **[3 marks]**

c. Differentiate between a Graph and a Tree and a spanning tree with an example in each case. **[3 marks]**

d. Graph A is represented by the following adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

i Draw the graph A. **[2 marks]**

ii Determine whether A is a tree. Justify your answer. **[2 marks]**

iii Determine whether A is Eulerian graph. Justify your answer. **[2 marks]**

QUESTION THREE **[20 MARKS]**

a. Differentiate between linear and non-linear recurrences. **[2 marks]**

b. Find the recurrence relation with initial condition for the following: 2, 10, 50, 250,. . . **[2 marks]**

c. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for n =2 , 3 , 4, . . . , and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3? **[2 marks]**

d. Solve $a_n - 3a_{n-1} = 2$, $n \geq 2$, with $a_0 = 1$. **[4 marks]**

e. Consider the second-order homogeneous recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 2$, $a_1 = 7$,

i Find the next three terms of the sequence. **[2 marks]**

ii Find the general solution. **[2 marks]**

iii Find the unique solution with the given initial conditions. **[2 marks]**

f. Solve the following recurrence relations:

$f_n = 10f_{n-1} - 25f_{n-2}$, where $f_0=3$ and $f_1=17$. **[4 marks]**

QUESTION FOUR [20 MARKS]

a. Define the following terms:

i Relatively prime [1 mark]

ii Modular arithmetic [1 mark]

b. Given as a=365 and b=211 find g(a, b)=s(a) + v(b) [5 marks]

c. Find a positive integer (a) such that when (a) is divided by 7 it gives a remainder of 4, when divided by 9 remainder is 5 and when divided by 11 remainder is 6. [5 marks]

d. Find the least positive values of x such that

i $84x-38\equiv 79 \pmod{15}$. [3 marks]

ii $78+x\equiv 3 \pmod{5}$ [2 marks]

iii $89\equiv(x+3)\pmod{4}$ [3 marks]

QUESTION FIVE [20 MARKS]

a. Define the following terms as used in the study of discrete structures

i Equiprobable Spaces [1 mark]

ii Random Variables [1 mark]

iii Independent Event [1 mark]

b. Define the expected value (or expectation) of the random variable X (s) on the sample space S. [2 marks]

What is the variance of the random variable X, where X is the number that comes up when a die is rolled? [3 marks]

c. Suppose a student is selected at random from 100 students where 30 are taking mathematics, 20 are taking chemistry, and 10 are taking mathematics and chemistry. Find the probability (p) that the student is taking mathematics or chemistry. [2 marks]

d. In a certain University, 25% of the students failed mathematics (M), 15% failed chemistry (C), and 10% failed both mathematics and chemistry. A student is selected at random.

i If he failed chemistry, find the probability that he also failed mathematics. [2 marks]

ii If he failed mathematics, find the probability that he also failed chemistry. [2 marks]

iii Find the probability that he failed mathematics or chemistry. [2 marks]

e. There are many lotteries now that award enormous prizes to people who correctly choose a set of six numbers out of the first n positive integers where n is usually between 30 and 60. What is the probability that a person picks the correct six numbers out of 40? **[4 marks]**