(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 428

COURSE TITLE: MATHEMATICAL MODELING

DATE: 7/10/2021 **TIME: 9:00 AM – 11:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1 (30 marks)

a. Define the following terms
   i. Model (2 marks)
   ii. Mathematical model (2 marks)

b. State four stapes in the mathematical modeling process (4 marks)

c. A 12-volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ Henry and the resistance is 10 ohms. Determine the current i if the initial current is 0 (6 marks)

d. Suppose the current Kenya population is 40 million and the birth rate is 0.02 and the death rate is 0.01
   i. What will be the population in ten years if the population keeps growing at the same rate (3 marks)
   iii. How many years will it take for the population to double its initial size? (3 marks)
   iv. How many births will occur between $t = 10$ and$t = 11$? (2 marks)

e. A body cools from 90^0c to 70^0c in 3 minutes at surrounding temperature of15^0c. Determine how long it will take for the body to cool to 50^0c. (6 marks)

f. State the Kirchhoff's 2nd law (2 marks)

Question 2(20 marks)

a. An enzymatic reaction involves a substance S reacting with an enzyme E to form a complex SE which in turn is converted into product P as shown below$S + E$ the enzyme is then free to participate in another reaction. Develop a model for the above reactions (8 marks)

b. Develop a mathematical model for measles and formulate it, explain what each and every term represents (12 marks)

Question 3 (20 marks)

Explain the predator prey model and include the lotka Volterra predator pray equation (20 marks)

Question 4 (20 marks)

a. find the solution of the differential equation
   $\frac{dp}{dt} = p(r - \frac{r}{k}p)$ (7 marks)

b. Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number x of the infected students but also to the number of students not infected. Use the solution in (a) to determine the number of infected students after 6 days if its further observed that the number of infected students after 4 days is 50. (6 marks)

c. when a cake is removed from oven its temperature is measured at 300°F, 3 minutes laither its temperature is 200°F how long will it take for the cake to cool to a room temperature 70°F (7 marks)

Question 5 (20 marks)

a. find the displacement of the spring after time t described by the system of differential equation

$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{r}{m} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} f(t)$ .in the case where the mass on the string is 1, the damping constant$r = 5$, the spring 3constant$k = 4$, the forcing function $f(t) = t$ and the initial extension of the spring is 0 with 0 initial velocity. (9 marks)

b. An electrical circuit contains inductance L and resistance R connected to a constant voltage source E. The current i is given by the differential equation

$E - L\frac{di}{dt} = Ri$ Where L and R are constants. Find the current in terms of t given that when $t = 0, i = 0$ (5 marks)

c. study the stability of the following system (6 marks)

$$0.001x(500 - 4x - y) = 0$$
$$0.001y(400 - y - 2x) = 0$$