(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 121**

COURSE TITLE: **ALGEBRAIC STRUCTURES I**

DATE: 24/9/2021 **TIME:** 8:00 A.M - 10 A.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i.	Trivial subgroup	(1mark)
ii.	Cyclic group	(2marks)
iii.	Bijective function	(2marks)
iv.	Field	(2marks)
v.	Inverse of a function	(2marks)
vi.	Union of sets	(2marks)

b) If S is a subset of the group G, show that s is a subgroup of G if and only if S is nonempty and whenever $a, b \epsilon S$,then $ab^{-1} \epsilon S$ (4 marks)

c) Show that every cyclic group is abelian (3marks)

d) If A is an invertible matrix, show that its inverse is unique (5marks)

e) Given the set $\mathbb{Z} \geq 3$, state the distinct cosets of $< 3 >$ in $\mathbb{Z}$ (4marks)

f) Show that G is cyclic if $|G| = p$ is a prime (3marks)

QUESTION TWO (20 MARKS)

a) Define the following

i.	Subgroup	(2marks)
ii.	Lagranges theorem	(2marks)
iii.	Bijective functions	(2marks)
iv.	Symmetric group	(2marks)

b) Draw the cayley table for the quaternion group (8marks)

c) Show that cosets are either identical or disjoint (4marks)

QUESTION THREE (20 MARKS)

a) Define the Klein four group K_4 and proof that it's an abelian group. (8marks)

b) Generate a 3×3 circulant matrix starting with $[a, b, c]$ (3marks)

c) Show that every subgroup of an abelian group is normal (3marks)

d) Given the set $S_3 \geq < (123) >$, state the distinct cosets of $< (123) >$ in S_3 (4marks)

e) Define a binary operation (2marks)

QUESTION FOUR (20 MARKS)

a) Define the following

   i. Simple group (2marks)

   ii. Normal subgroup (2marks)

   iii. Quotient group (2marks)

   iv. Index of a group (2marks)

b) Find the inverse of the following matrix, whose entries are elements of $\mathbb{Z}_6$ (6marks)

$$A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$$

c) Let $H \leq G$ and $x, y \in G$ then proof that either $xH = yH$ or $xH \cap yH = \emptyset$ (6marks)

QUESTION FIVE (20 MARKS)

a) Determine the symmetric group s_3 (7marks)

b) Define the following

   i. Group (4marks)

   ii. Ring (5marks)

c) State 4 examples of fields (4marks)