KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
(MAIN EXAMINATIONS)
2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS
FOR THE
DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MAP 122

COURSE TITLE: LINEAR ALGEBRA I

DATE: 22/07/2021 **TIME: 2PM-4PM**

INSTRUCTIONS TO CANDIDATE

- Answer question ONE (COMPULSORY) and any other TWO questions

DURATION: 2 HOURS

QUESTION ONE (COMPULSORY) [30 marks]

a) Let V be a vector space over a field F. given that U= $\{u_1, u_2, \ldots, u_n\}$ is a subset of V, explain the meaning of the following; U is

i. a subspace of V (2 mks)

ii. linearly independent (2 mks)

iii. a basis for V (2 mks)

b) consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 0 \end{pmatrix}$$

Find the reduced row echelon equivalence of A. (3 mks)

c) solve the system

$$x_2 + x_3 - 2x_4 = -3$$

$$x_1 + 2x_2 - x_3 = 2$$

$$2x_1 + 4x_2 + x_3 - 3x_4 = -2$$

$$x_1 - 4x_2 - 7x_3 - x_4 = -19$$

(10 mks)

d) find the kernel of the linear transformation T: $R^2 \rightarrow R^3$ represented by

$$T(x_1, x_2) = (x_1 - 2x_2, 0, -x_1)$$

(5 mks)

e) Let T: $R^5 \rightarrow R^7$ be a linear transformation

i. Find the dimension of the kernel of T if the dimension of the range is 2 (2 mks)

ii. Find the rank of T if the nullity of t is 4 (2 mks)

iii. Find the rank of T if ker (T) = {0} (2 mks).

QUESTION TWO (20 marks)

(a) If W is a nonempty subset of a vector space V, then W is a subspace of V iff the following closure conditions hold.

If u and v are in W, then u+v is in W.

If u is in W and c is a scalar, then cu is in W. Prove (6 mks)

(b) Let W be the set of all 2x2 symmetric matrices. Show that W is a subspace of the vector space $M_{2,2}$, with the standard operations of matrix addition and scalar multiplication. (4 mks)

(c) If V and W are both subspaces of a vector space U, then the intersection of V and W is also a subspace of U. (6 mks)

(d) Show that the subset of R^2 consisting of all points on the unit circle $x^2+y^2 = 1$ is not a subspace. (4 mks)

QUESTION THREE (20 marks)

a. Consider the set of vectors $V = \{[x, y, z] : ax + by + cz = 0\}$ where a, b, c are scalars. Show that V is a vector space. (10 mks)

b. Let V be a vector space, then

c. $\alpha.0 = 0$ for every scalar α

d. $0.x = 0$ for every x in V

e. If $\alpha.x = 0$ then $\alpha = 0$ or $x = 0$ (10 mks)

QUESTION FOUR (20 marks)

a. Show that the set of vectors

$$\begin{pmatrix}1\\1\\1\end{pmatrix} \begin{pmatrix}0\\1\\1\end{pmatrix} \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ spans } R^3$$ (5 mks)

b. If A and B are invertible matrices of size n, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$ (6 mks)

c. Determine whether the set of vectors in P_2 is linearly independent or linearly dependent $S = \{1 + x - 2x^2, 2 + 5x - x^2, x + x^2\}$ (9 mks)

QUESTION FIVE (20 marks)

a. When is an nxn matrix A invertible? (2 mks)

b. If A is an invertible matrix, then its inverse is unique. Prove (7 mks)

c. Show that B is the inverse of A where,

$A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}$ (5 mks)

d. Compute A^{-1} in two ways and show that the results are equal given that

$A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$ (6 mks)