(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 869**

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 14/10/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

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QUESTIONONE [20 MARKS]

(a) State the following terms
  (i) Riemann mapping theorem (2 mks)
  (ii) Conformal mapping (2 mks)

(b) Find the Laurent series about the indicated singularity for the function
$f(z) = \frac{1}{z^2-3z+2} \qquad z < 1$ (4 mks)

(c) Determine the linear fractional transformation that maps $z = 1,0,-1$ onto $w = i, \infty, 1$ respectively (6 mks)

(d) Evaluate $\oint_C (z - Re(z))dz \qquad C\colon |z| = 2$ (6 mks)

QUESTION TWO [20 MARKS]

Consider the triangle $\boldsymbol{A(0,0)}$, $\boldsymbol{B(2,0)}$ and $\boldsymbol{C(2,2)}$
  (i) Draw the triangle and its image under $T(z) = (4+5i)z - (6+2i)$ (12 mks)
  (ii) Discuss conformity of T at $A(0,0)$ and $C(2,2)$ (8 mks)

QUESTION THREE [20 MARKS]

(a) If $f(z) = z^5 - 2z^3 + 3z + 2 - i$, evaluate
$\int_C \frac{f'(z)}{f(z)} dz$ where C encloses all zeros of $f(z)$ (4 mks)

(b) Show that $cot^{-1}(z) = \frac{1}{2i} ln\left(\frac{z+i}{z-i}\right)$ (5 mks)

(c) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$
  (i) Along the parabola $x = 2t$, $y = t^2 + 3$ (5 mks)
  (ii) Straight lines from (0,2) to (3,2) and then (3,2) to (3,4) (6 mks)

QUESTION FOUR [20 MARKS]

(a) Find the residuals of the function $\boldsymbol{f(z) = \frac{z^3+2}{(z^2+4)^2}}$ (5 mks)

(b) Evaluate $\oint_C \frac{e^{2z}}{(z-1)^5} dz$ where C is a circle $|z| = 3$ (5 mks)

(c) Evaluate $\oint_C (6x + 5y + 7)dx + (4x - 3y - 2)dy$ around a triangle in the xy plane with vertices at (0,0), (3,0) and (3,2) (5 mks)

(d) Determine the number of zeros of $z^5 - 6z^2 + z - 1$ interior to $|z| = 1$ (5 mks)

QUESTION FIVE [20 MARKS]

(a) Prove that the function $f_1(z) = \int_0^\infty 2t^3 e^{-zt} dt$ is analytic at all points of z for which $Rez > 0$ (6 mks)

(b) State and prove the Rouche's theorem (14 mks)