(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 224/MAA 213

COURSE TITLE: ANALYTIC GEOMETRY

DATE: 7/10/2021 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms:

i) A plane curve (1 mark)

ii) Direction cosines (1 mark)

iii) A conic (1 mark)

iv) Ratio (1 mark)

b) Find the length of the arc $y = \ln \sec x$ between $0 \leq x \leq \frac{\pi}{4}$ (5 marks)

c) Write the line L through the point $P(3,4,5)$ and parallel to the vector $\underset{\sim}{V} = (5,-2,7)$ in

i) Vector form (1mark)

ii) Parametric form (1 mark)

iii) Symmetric form (1 mark)

d) Find the equation of the line through $(7,5)$ perpendicular to the line $4x - 3y = 1$ (4 marks)

e) Find the perpendicular distance of the point $P(0,14,10)$ from the line whose equation is $\underset{\sim}{r} = \left(\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}\right) + \lambda\left(3\underset{\sim}{i} + 4\underset{\sim}{k}\right)$

(5 marks)

f) Find the length of the curve $x^3 = y^2$ between $x = 0$ and $x = 1$. (5 marks)

g) Find the equation of the plane with a normal vector $\underset{\sim}{LM} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$ and passing through the point $B(3,1,-1)$ (4 marks)

QUESTION TWO (20 MARKS)

a) Find the point of intersection of the plane $3x - y - 2z = 7$ and the line:

$$\frac{x+3}{5} = \frac{y+1}{2} = \frac{z+4}{3}$$

(4 marks)

b) Find the coordinates of the point where the line through $(5,1,6)$ and $(3,4,1)$ crosses the plane $zx-$plane (4 marks)

c) Given lines whose direction ratios are given by the relations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$, find the angle between the lines. (5 marks)

d) Given that $R(7,1,2)$, $S(3,-1,4)$, $T(4,-2,5)$:

(i) Show that $SR \perp ST$ (4 marks)

(ii) Hence obtain the equation of the plane through T perpendicular to SR. (3 marks)

QUESTION THREE (20 MARKS)

a) Convert $\left(-1,1,-\sqrt{2}\right)$ from the Cartesian to spherical coordinates. (4 marks)

b) Sketch the conic $9x^2-4y^2-72x+8y+176=0$ and find its centre, foci and the asymptotes (5 marks)

c) Find the equation of the tangent to the ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $(a\cos\theta, b\sin\theta)$. (4 marks)

d) Find the coordinates of the foci, the lengths of the major and minor axes and sketch the graph: $2x^2+y^2=10$ (4 marks)

e) Show that the equation $y=5x-2x^2$ represents a parabola and find the length of its latus rectum (3 marks)

QUESTION FOUR (20 MARKS)

a) Find the:

(i) ratio in which the line through the points $(1,-3,2)$ and $(-5,4,-3)$ is divided by the plane $2x-3y+z+6=0$ (3 marks)

(ii) coordinates of the point of intersection (2 marks)

b) Find the length of the curve $y=10\cosh\frac{x}{10}$, between $x=-1$ and $x=2$ (4 marks)

c) Find the parametric equation of the line of intersection of the planes

$3x-y+4z-7=0$ and $x+y-2z+5=0$ (4 marks)

d) Obtain the equation of the plane with normal vector $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ containing the Point $(4,0,-3)$ (3 marks)

e) What are the direction cosines of a line normal to the lines whose equations are given by $\frac{x-4}{-1}=\frac{y+3}{2}=\frac{z-5}{-3}$ and $\frac{x+4}{-2}=\frac{y+1}{1}=\frac{z-2}{3}$ (4 marks)

QUESTION FIVE (20 MARKS)

a) Find where the line joining the points $(0,1,0)$ and $(1,0,1)$ meets the plane $x-y+z=1$ (3 marks)

b) Find the length of the arc from $\theta=0$ to $\theta=\alpha$ of the curve given by $x=a\cos\theta,\ y=a\sin\theta$. (3 marks)

c) Find the equation of the plane containing the points $F(6,-7,-3)$, $G(3,-3,2)$ and $H(7,4,2)$ (3 marks)

d) Find the locus of the point whose distance from the point $(2,-2,2)$ is two times its distance from the plane $2x+3y-6z=2$ (4 marks)

e) Determine the direction cosines and direction angles for $\underset{\sim}{a}=(2,1,-4)$ (3 marks)

f) Find c in terms of a,b,m if $y=mx+c$ is a tangent to the hyperbola:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$ (4 marks)

END