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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAT 224/MAA 213

COURSE TITLE: ANALYTIC GEOMETRY

DATE: 7/10/2021

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define the following terms:
- i) A plane curve (1 mark)
 - ii) Direction cosines (1 mark)
 - iii) A conic (1 mark)
 - iv) Ratio (1 mark)
- b) Find the length of the arc $y = \ln \sec x$ between $0 \leq x \leq \frac{\pi}{4}$ (5 marks)
- c) Write the line L through the point $P(3,4,5)$ and parallel to the vector $\underline{V} = (5, -2, 7)$ in
- i) Vector form (1 mark)
 - ii) Parametric form (1 mark)
 - iii) Symmetric form (1 mark)
- d) Find the equation of the line through $(7,5)$ perpendicular to the line $4x - 3y = 1$ (4 marks)
- e) Find the perpendicular distance of the point $P(0,14,10)$ from the line whose equation is $\underline{r} = (\underline{i} + 2\underline{j} + 3\underline{k}) + \lambda(3\underline{i} + 4\underline{k})$ (5 marks)
- f) Find the length of the curve $x^3 = y^2$ between $x = 0$ and $x = 1$. (5 marks)
- g) Find the equation of the plane with a normal vector $\underline{LM} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$ and passing through the point $B(3,1,-1)$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Find the point of intersection of the plane $3x - y - 2z = 7$ and the line: $\frac{x+3}{5} = \frac{y+1}{2} = \frac{z+4}{3}$ (4 marks)
- b) Find the coordinates of the point where the line through $(5,1,6)$ and $(3,4,1)$ crosses the plane $zx - plane$ (4 marks)
- c) Given lines whose direction ratios are given by the relations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$, find the angle between the lines. (5 marks)
- d) Given that $R(7,1,2)$, $S(3,-1,4)$, $T(4,-2,5)$:
- (i) Show that $\underline{SR} \perp \underline{ST}$ (4 marks)
 - (ii) Hence obtain the equation of the plane through T perpendicular to \underline{SR} . (3 marks)

QUESTION THREE (20 MARKS)

- a) Convert $(-1, 1, -\sqrt{2})$ from the Cartesian to spherical coordinates. (4 marks)
- b) Sketch the conic $9x^2 - 4y^2 - 72x + 8y + 176 = 0$ and find its centre, foci and the asymptotes (5 marks)
- c) Find the equation of the tangent to the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a \cos \theta, b \sin \theta)$. (4 marks)
- d) Find the coordinates of the foci, the lengths of the major and minor axes and sketch the graph: $2x^2 + y^2 = 10$ (4 marks)
- e) Show that the equation $y = 5x - 2x^2$ represents a parabola and find the length of its latus rectum (3 marks)

QUESTION FOUR (20 MARKS)

- a) Find the:
- (i) ratio in which the line through the points $(1, -3, 2)$ and $(-5, 4, -3)$ is divided by the plane $2x - 3y + z + 6 = 0$ (3 marks)
- (ii) coordinates of the point of intersection (2 marks)
- b) Find the length of the curve $y = 10 \cosh \frac{x}{10}$, between $x = -1$ and $x = 2$ (4 marks)
- c) Find the parametric equation of the line of intersection of the planes $3x - y + 4z - 7 = 0$ and $x + y - 2z + 5 = 0$ (4 marks)
- d) Obtain the equation of the plane with normal vector $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ containing the Point $(4, 0, -3)$ (3 marks)
- e) What are the direction cosines of a line normal to the lines whose equations are given by $\frac{x-4}{-1} = \frac{y+3}{2} = \frac{z-5}{-3}$ and $\frac{x+4}{-2} = \frac{y+1}{1} = \frac{z-2}{3}$ (4 marks)

QUESTION FIVE (20 MARKS)

- a) Find where the line joining the points $(0, 1, 0)$ and $(1, 0, 1)$ meets the plane $x - y + z = 1$ (3 marks)
- b) Find the length of the arc from $\theta = 0$ to $\theta = \alpha$ of the curve given by $x = a \cos \theta$, $y = a \sin \theta$. (3 marks)

- c) Find the equation of the plane containing the points $F(6,-7,-3)$, $G(3,-3,2)$ and $H(7,4,2)$ (3 marks)
- d) Find the locus of the point whose distance from the point $(2,-2,2)$ is two times its distance from the plane $2x + 3y - 6z = 2$ (4 marks)
- e) Determine the direction cosines and direction angles for $\underline{a} = (2,1,-4)$ (3 marks)
- f) Find c in terms of a, b, m if $y = mx + c$ is a tangent to the hyperbola:
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
 (4 marks)

END