(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 222/MAA 221

COURSE TITLE: CALCULUS III

DATE: 11/05/2022 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i) Local maximum point (2 mks)
   (ii) Critical point (2 mks)
   (iii) Saddle point (2 mks)

b) Find the domain and range for the function $f(x,y) = \frac{2}{x^2-y}$ (2 mks)

c) The production function is given by $f(x,y) = 2xy + x^2$ maximize the this function subject to budget constraint $x - 2y = 16$ (5 mks)

d) Find the volume in the 1st octant between the planes $z = 0$, and $z = 2x + 3y - 5$ and inside the cylinder $x^2 + y^2 - 9 = 0$ (5 mks)

e) Investigate the convergence of $\sum_{k=0}^{\infty} \frac{(-5)^{2k} e^{2k}}{2k!}$ (5 mks)

f) Locate any relative extreme points and determine their nature for the function $f(x,y,z) = 3x^2 + 5y^2 + z^2 - 16x + 15y - 4z - 10$ (7 mks)

QUESTION TWO (20 MARKS)

a) Use the 1st principles to determine $\frac{\partial f}{\partial y}$ given that $f(x,y) = 6xy - 2x^2y^2 - y^3$ (4 mks)

b) Let $f(x,y,z) = x\ln(xz) - e^{2x^2y} + 3\sin(xyz)$. Find
   (i) f_{xy} (2 mks)
   (ii) f_{yxz} (3 mks)

c) Evaluate $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\sin y} 2\sin z\cos y\,dx\,dy\,dz$ (5 mks)

d) Verify that the Tailor series expansion for the function $f(x) = \sin x$ about $x = 0$ is $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$ hence find the Maclaurin series for $f(x) = 3x\sin x$ (6 mks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim_{(x,y)\to(2,2)} \frac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ (4 mks)

b) Suppose z is a differentiable function near each (x,y) for the equation $x\cos y - 4y^2z + \ln(xyz) = 6$ find $\frac{\partial z}{\partial y}$ (3mks)

c) An open cylinder has a volume of $112.32\ cm^3$ Find the radius and the height that will yield maximum surface area (6 mks)

d) Use the Lagrange multipliers to find the local extrema of the function $f(x,y) = x^3 + 3y^2$ Subject to $x^2 + y^2 = 16$ (7 mks)

QUESTION FOUR (20 MARKS)

a) If $R = \{x, y \,/\, 1 \leq x \leq 2 \text{ and } 0 \leq y \leq 3\}$ evaluate $\iint_R (26x^2y + y - 8x)dA$ (3 mks)

b) Let $z = e^{-3x} \sin y$ and $x = 2st^2 - 3t$ and $y = t^3 - 3s$ find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ (7 mks)

c) Find the volume of the solid bounded by the graphs of $z = 4 - y^2$, $x - z = 3, x = 0$, and $z = 0$ (5 mks)

d) Consider the series $S_n = \frac{1}{\sqrt{2n-1}}$ using the integral test, determine whether the series converges or diverges (5 mks)

QUESTION FIVE (20 MARKS)

a) Consider the series $\sum_{k=1}^{\infty} \frac{(-4)^{k-1} 6^k}{2k^2}$ use ratio theorem to show that the series diverges (5 mks)

b) Find the radius and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(-3)^k (x-5)^k}{k 2^{k-1}}$ (5 mks)

c) Locate and classify all critical points of $f(x_1, x_2, x_3) = 4x_2 + 12x_1x_3 - 4x_1^3 - 2x_2^2 - 12x_3^2 + 2$ (10 mks)