(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAA 223

COURSE TITLE: CLASSICAL MECHANICS

DATE: 13/05/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) State three important properties of a central force field (3 marks)

b) The moon revolves about the earth in $30 days$. Assuming that the orbit is circular and has a radius of $4.0 \times 10^5 km$, calculate the acceleration of the moon towards the earth. (4 marks)

c) Find the constant force needed to accelerate a mass of $10kg$ moving along a straight line from a speed of $54km/h$ to a speed of $108km/h$ in 5 seconds. (6 marks)

d) An electric motor rated $3kW$ is used to lift bales of hay to a store in a dairy farm. A single bale has a mass of $5kg$. If the store is $5m$ above the ground, how many bales can the motor raise in 3 minutes? (5 marks)

e) The three motion laws of Newton are the actions of mechanics. State these laws (3 marks)

f) A particle of mass 2 units moves in a force field $\vec{F}$ depending on time t given by $\vec{F} = 24t^2 \underset{\sim}{i} + (36t - 16)\underset{\sim}{j} - 12t \underset{\sim}{k}$. Assuming that at $t = 0$, the particle is located at $\vec{R}_0 = 3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ and has a velocity $\vec{V}_0 = 6\underset{\sim}{i} + 16\underset{\sim}{j} - 8\underset{\sim}{k}$, find

(i) Velocity at any time t (5 marks)

(ii) Position at any time t (4 marks)

QUESTION TWO (20 MARKS)

a) Consider two blocks A and B of mass m_A and m_B connected by a light spring on a frictionless surface. If the masses are pulled apart so that the spring is stretched after which they are released, show that:

(i) The velocity of A is given by $\vec{V}_A = -\vec{V}_B\left(\frac{m_B}{m_A}\right)$ (5 marks)

(ii) The ratio of their kinetic energies is given by $\frac{K_A}{K_B} = \frac{m_B}{m_A}$ (5 marks)

b) Given that the acceleration due to gravity on the surface of the moon is $1.7ms^{-2}$ and the radius of the moon is $1.70 \times 10^6 m$. Calculate the mass of the moon. (5 marks)

c) A trolley M of mass $80g$ travelling at $3ms^{-1}$ collides with another trolley N of twice its mass moving in the opposite direction with a velocity of $0.5ms^{-1}$. If the trolleys stuck together on collision, calculate the common velocity with which they move. (5 marks)

QUESTION THREE (20 MARKS)

g) A particle of mass m moves in the $xy-plane$ so that its position vector is given by $r = a\cos\omega t\, i + b\sin\omega t\, j$ where a, b and ω are positive constants and $a > b$

(1) Show that the:
   (i) particle moves in an ellipse (5 marks)
   (ii) force acting on the particle is always directed towards the origin (4 marks)

(2) Find the kinetic energy of the particle at points P and Q (5 marks)

(3) Find the work done by the force field in moving the particle from P to Q (3 marks)

h) What is meant by the terms:
   i) Parking orbit (1 mark)
   ii) Escape velocity (1 mark)
   iii) Impulse (1 mark)

QUESTION FOUR (20 MARKS)

a) Find the work done in moving a particle along a straight line from $(-4,3,2)$ to $(2,-1,3)$ in a force field given by $\vec{F} = 3i - 2j + k$ (3 marks)

b) Due to a force field a mass of $5units$ moves along a space curve whose position vector is $r = (2t^3 + t)i + (3t^4 - t^2 + 8)j - 12t^2 k$. Find

   (i) Velocity (2 marks)
   (ii) Momentum (2 marks)
   (iii) Acceleration (2 marks)
   (iv) Force at any time t (2 marks)

c) Given that the mass of the earth is $6\times10^{24}\,kg$ and the gravitational constant is $6.7\times10^{-11}\,m^3kg^{-1}s^{-2}$. The radius of the earth is $6.4\times10^6\,m$. Calculate the gravitational force on a mass of $5kg$ on the earth's surface. (4 marks)

d) An artificial satellite is rotating about the earth at a height h of $200km$ above the surface of the earth where acceleration due to the gravity $g = 9.8ms^{-2}$. The radius of the earth r is $6400km$. Calculate the speed of the satellite. (5 marks)

QUESTION FIVE (20 MARKS)

a) A constant force $\vec{F}$ acting on a particle of mass m changes the velocity from v_1 to v_2 in time τ. Prove that $\vec{F} = \frac{m(v_2 - v_1)}{\tau}$ (7 marks)

b) As an example of recoil, consider a radioactive decay in which an alpha particle, the nucleus of Helium atom of mass number 4 and atomic number 2, is emitted from a Uranium-238 nucleus originally at rest with a speed of $1.4 \times 10^7 m/s$ and kinetic energy of $4.1MeV$. Find

(i) the velocity of the residual Thorium-234 nucleus. (5 marks)

(ii) kinetic energy of Thorium (4 marks)

c) Find the work done in moving a particle along the vector $\vec{S} = 5\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ if the force applied is $\vec{F} = 3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ (4 marks)