(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 432**

COURSE TITLE: METHODS II

DATE: 1/10/2021 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

(a) Given $f(x,y) = sinx + cosy$, use implicit function theorem to evaluate $\frac{dy}{dx}$ (7 marks)

(b) Find the equation of the tangent to the curve $r = sin\theta\ at\ \theta = \frac{\pi}{3}$ (6 marks)

(c) Given $f(t) = sint\ and\ h(t) = t$, find the convolution of $f(t) and\ h(t)$ (6 marks)

(d) Evaluate $\oint_C (x^2 - y^2)dx + (2y - x)dy$ where C consists of the region in the first quadrant that is bounded by the curves $y = x^2\ and\ y = x^3$ (7 marks)

(e) Evaluate $\int_1^2 \int_0^y (8xy + 1)dxdy$ (4 marks)

QUESTION TWO (20MARKS)

(a) If $z = x^2 + y^2\ where\ x = rcos\emptyset\ and\ y = rsin\ \emptyset, show\ that$

$\frac{\partial z}{\partial r} = 2xcos\emptyset + 2ysin\emptyset$ (7 marks)

(b) $Given\ f = 3x^2 + 2xy + 4y^2, show\ that\ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ (7 marks)

(c) Given that $U = x^3 - xy - y^4$, find the value of $\frac{\partial^2 U}{\partial y^2}$ if $x = -1\ and\ y = -2$

(6 Marks)

QUESTION THREE (20 MARKS)

(a) If $g(t) = e^{-3t}\ and\ h(t) = e^{2t}, show\ that\ g * h(t) = h * g(t)$ (7 marks)

(b) Evaluate $L\{\int_0^t e^u \sin(t-u)\,du\}$ (6 marks)

(c) Show that $\int_0^\pi \int_0^a 2\pi r^2 sin\theta d\theta dr = \frac{2\pi r^4}{3}(1 - cosa)$ (7 marks)

QUESTION FOUR (20 MARKS)

(a) Convert (i) (4,-3) to polar coordinates

(ii) (4.5, 5.16 rads) to cartesian form (6 marks)

(b) (i) Convert the polar equation $r = -3cos\theta$ to rectangular form

(ii) Convert $xy = 4$ to polar form simplifying your answer (7 marks)

(c.) Find the gradient of the tangent to the curve $r = \theta$ when $\theta = \frac{\pi}{2}$ (7 marks)

QUESTION FIVE (20 MARKS)

(a) Evaluate $\int_0^2 \int_{-1}^2 \int_1^3 (x + y^2 + z)dxdzdy$ (6 marks)

(b) Compute the line integral $\oint_C (5 - xy - y^2)dx - (2xy - x^2)dy$, where C is the boundary of the square $R = \{(x,y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$ (7 marks)

(c) Use Green's theorem to evaluate $\oint_C x^2ydx + x^2dy$, where C is the boundary, described anticlockwise, of the tringle whose vertices are (0,0), (1,0), (1,1) (7 marks)