(*Knowledge for Development)*

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 112/STA 142**

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 27/9/2021 **TIME:** 11 AM – 1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: Sample outcome, Universal set, Mutually exclusive events (3 mks)

   (b) For two events A and B, suppose that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \bigcup B) = 0.6$. Calculate $P(A \bigcap B)$. (3 mks)

   (c) A woman has her purse snatched by two teenagers. She is subsequently shown a police lineup consisting of five suspects, including the two perpetrators. What is the sample space associated with the experiment "Woman picks two suspects out of lineup"? Which outcomes are in the event A: She makes at least one incorrect identification? (3 mks)

   (d) i. Explain the following: discrete random variable and continuous random variable (2 mks)

   ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	2a	3a	4a	5a	6a	7a	8a	9a

   A. Determine the value of a (2 mks)

   B. Find $P(X < 3)$, $P(0 < X < 5)$ (4 mks)

   (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (4 mks)

   (f) In a Bernoulli experiment,the first outcome has a probability x to occur, the second probability x^2. What is x? (3 mks)

   (g) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3." Calculate $P(A|B)$; Are A and B independent? (3 mks)

   (h) If two fair dice are rolled once, what is the probability that the total number of spots shown is equal to 5? (3 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0, & elsewhere \end{cases}$$

Compute;

i. $E(X)$, (2 mks)

ii. $E(3X)$ (3 mks)

iii. $Var(X)$ (3 mks)

(b) Suppose A and B be two events defined on a sample space S such that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \bigcup B) = 0.7$. Find;

i. $P(A \bigcap B)$ (2 mks)

ii. $P(A^c \bigcup B^c)$ (2 mks)

iii. $P(A^c \bigcap B)$ (2 mks)

(c) Students in a certain university subscribe to three news magazines A, B, and C according to the following proportions: A : 20%, B : 15%, C : 10%, both A and B : 5%, both A and C : 4%, both B and C : 3%, all three A, B, and C : 2%. If a student is chosen at random, what is the probability he/she subscribes to none of the news magazines? (6 mks)

QUESTION THREE (20 MARKS)

3. (a) A university library has five copies of a textbook to be used in a certain class. Of these copies, numbers 1 through 3 are of the 1st edition, and numbers 4 and 5 are of the 2nd edition. Two of these copies are chosen at random to be placed on a 2-hour reserve.

i. Write out an appropriate sample space S. (2 mks)

ii. Consider the events A, B, C, and D, defined as follows, and express them in terms of sample points.

A = both books are of the 1st edition, (2 mks)

B = both books are of the 2nd edition, (2 mks)

C = one book of each edition, (2 mks)

D = no book is of the 2nd edition. (2 mks)

(b) Let X have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find

i. $Var(X)$ (5 mks)

ii. $Var(5X)$ (5 mks)

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y. (8 mks)

   (b) Let X (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} cx, & 0 \leq x \leq 3 \\ c(10-x), & 3 < x \leq 5 \\ 0, & elsewhere \end{cases}$$

   i. Find c such that $f(x)$ is a pdf (4 mks)

   ii. Find $P(X \leq 3)$ (2 mks)

   iii. Find $P(X > 3)$ (2 mks)

   iv. Find $P(2.5 \leq X \leq 5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Two boxes each contain three cards. The first box contains cards labeled 1, 3 and 5. The second box contains cards labeled 1, 3, and 5. In a game, a player draws one card at random from each box and his score, X, is the sum of the numbers on the two cards.

   i. Obtain the six possible values of X and find their corresponding probabilities (2 mks)

   ii. Calculate the standard deviation of X. (8 mks)

   (b) The faculty in an academic department in Kibabii University consists of 4 assistant professors, 6 associate professors, and 5 full professors. Also, it has 30 graduate students. An ad hoc committee of 5 is to be formed to study a certain curricular matter.

   i. What is the number of all possible committees consisting of faculty alone? (3 mks)

   ii. How many committees can be formed if 2 graduate students are to be included and all academic ranks are to be represented? (4 mks)

   iii. If the committee is to be formed at random, what is the probability that the faculty will not be represented? (3 mks)