(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

YEAR THREE SEMESTER TWO MAIN EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: STA 325

COURSE TITLE: MULTIVARIATE PROBABILITY DISTRIBUTION

DATE: 7/10/2021 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer **Questions ONE** and **ANY OTHER TWO**.

QUESTION ONE [30 MARKS]

(a) Explain the following terms

(i) Principle component analysis (1mk)

(ii) Random vector (1mk)

(iii) Multivariate data (1mk)

(b) Given the joint pdf of random variables X, Y and Z as

$$f(x,y,z) = \begin{cases} e^{-x-y-z}\,; 0 < x < \infty, 0 < y < \infty, 0 < z < \infty \\ 0\,; elsewhere \end{cases}$$

Find the joint cumulative distribution function (cdf) of the three random variables. (5mks)

(c) Let $\underline{x} = [1, 3, 2]$ and $\underline{y} = [-2,\ 1, -1]$ find

(i) The length of $\underline{x}$ (1mk)

(ii) The angle between $\underline{x}$ and $\underline{y}$ (3mks)

(iii) The length of the projection of $\underline{x}$and $\underline{y}$ (1mk)

(d) Let $A = \begin{bmatrix} 3 & -\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$

(i) Is A symmetric? Give reason (1mk)

(ii) Show that A is positive definite (4mks)

(e) Consider the following $n = 7$ observations on $p = 2$ variables

x_1	3	4	2	6	8	2	5
x_2	5	5.5	4	7	10	5	7.5

(i) Compute the sample means $\bar{x}_1$ and $\bar{x}_2$ and the sample variances S_{11} and S_{22} (4mks)

(ii) Compute the sample covariance S_{12} and the sample correlation coefficient r_{12} and interpret these quantities (5mks)

(iii) Display the sample mean array $\bar{x}$, the sample correlation array R and the sample variance-covariance S_{12} (3mks)

QUESTION TWO [20 MARKS]

(a) In an experiment involving two correlated variables, the following sample statistics were obtained: $\underline{\overline{X}} = \begin{bmatrix} 10.00 \\ 10.00 \end{bmatrix}$ $S = \begin{bmatrix} 0.7986 & 0.6793 \\ 0.6793 & 0.7343 \end{bmatrix}$. Determine

(i) Principal components (8mks)

(ii) Variance of each principal component (3mks)

(iii) Percentage of variance explained by each principal component (3mks)

(b) Let $\underline{x}$ be a p-variate random vector, A be a non-zero matrix constants and $\underline{b}$ a $p \times 1$ vector of constants, show that

$$var(A\underline{x} + \underline{b}) = A\Sigma A'$$ (6mks)

QUESTION THREE [20 MARKS]

a) Let three random variables have the joint probability density function (pdf) as follows:

$$f(x_1, x_2, x_3) = \begin{cases} 8x_1x_2x_3; & 0 < x_1 < 1, 0 < x_2 < 1, 0 < x_3 < 1 \\ 0, & otherwise \end{cases}$$

Compute the expected value $5X_1X_2^3 + 3x_2X_3^4$ of (5mks)

b) Use the pdf in (b) above to compute $\Pr(X_1 \leq \frac{1}{2}, X_2 \leq \frac{1}{2}, X_3 \leq \frac{1}{2})$. (5mks)

c) Assume $\underline{x}' = (x_1, x_2, x_3)$ is normally distributed with mean vector $\underline{\mu} = (1, -1, 2)$ and variance matrix $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. Find the distribution of $3x_1 - 2x_2 + x_3$ (5mks)

d) Find the maximum likelihood estimators of the mean vector $\underline{\mu}$ and covariance matrix Σ based on the data matrix

$$x = \begin{bmatrix} 5 & 1 \\ -2 & 3 \\ 3 & 4 \end{bmatrix}$$ (5mks)

QUESTION FOUR [20 MARKS]

a) Let random variables X, Y and Z have the joint pdf given by

$$f(x,y,z)=\begin{cases}\dfrac{12x^{2}+12yz}{7}; & 0<x<1, 0<y<1, 0<z<1\\ 0, & otherwise\end{cases}$$

i) Use the joint pdf to find $f(z\backslash x,y)$. (3mks)

ii) What is the $E(Z\backslash x=\frac{1}{2}, y=\frac{1}{2})$? (5mks)

iii) Find $Var(Z\backslash x=\frac{1}{2}, y=\frac{1}{2})$. (5mks)

b) Show that the sample mean is an unbiased estimator of $\underline{\mu}$ and that the sample variance is biased estimator of matrix Σ (7mks)

QUESTION FOUR [20 MARKS]

(a) Define a random vector

(b) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma=\begin{bmatrix}1 & 1 & 2\\ 1 & 9 & -3\\ 2 & -3 & 25\end{bmatrix}$$

(i) Obtain the population correlation matrix (ρ) and $V^{\frac{1}{2}}$ (6mks)

(ii) Multiply your matrices to check the relation $V^{\frac{1}{2}}\rho V^{\frac{1}{2}}$ (3mks)

(c) Find the covariance matrix for the two random variables X_1 and X_2 when their joint probability function $P_{12}(x_1,x_2)$ is represented by the entries in the study of the following table

X_2 / X_2	0	1	$P_1(X_1)$
-1	0.24	0.06	0.3
0	0.16	0.14	0.3
1	0.40	0.00	0.4
$P_2(X_2)$	0.8	0.2	

(9mks)