(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAP 314**

COURSE TITLE: **NUMBER THEORY**

DATE: 14/07/21 **TIME: 9 AM -11 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (Compulsory)

a) Let $a, b, c, d, m \in \mathbb{Z}, k \in \mathbb{N}$ with $a \equiv b \ mod\, m$ and $c \equiv d \ mod \ m$. Show that

$$a + c \equiv b + d \ mod \ m, \qquad a - c \equiv b - d \ mod\, m,$$

$$ac \equiv bd \ mod\, m, \quad a^k \equiv b^k \ mod\, m$$

And if f is a polynomial with integral coefficients then $f(a) \equiv f(b) \ mod \ m$.

(5 marks)

b) Show that the set $A = \{-40,6,7,15,22,35\}$ forms a complete residue set $mod\ 6$ while the set $B = \{-3, -2, -1,1,2,3\}$ $mod\ 6$ does not. (5 marks)

c) Prove that the product of n consecutive integers is divisible by $n!$ and hence show that $n^5 - 5n^3 + 4n$ *is* always divisible by 120. (5 marks)

d) Find all the solutions in integers to $3456x + 246y = 234$. (5 marks)

e) State and prove the Euler's theorem (5 marks)

f) Find the Remainder when 6^{1987} is divided by 37 (5 marks)

QUESTION TWO

a) Prove that $\log_2 3$ is irrational. (5 marks)

b) Show that $n^2 + 23$ is divisible by 24 for infinitely many n . (5 marks)

c) State the Euclidean Algorithm process of finding greatest common divisor and use it to find the greatest common divisor 560 and 600. (5 marks)

d) Solve the congruence $50x \equiv 12 \ mod \ 14$ (5 marks)

QUESTION THREE

a) State and prove the Wilson's theorem. (5 marks)

b) Prove that $n^4 + 4$ is prime only when $n = 1$ for all $n \in \mathbb{N}$ (5 marks)

c) Prove by induction that $\sum_{t=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3}$,for all natural numbers $n \geq 2$

(5 marks)

d) find the last two digits of 3^{1000} (5 marks)

QUESTION FOUR

a) State the well-ordering axiomand use it to show that there is no integer between 0 and 1.

(5 marks)

a) . Solve the congruence $3x^2 + 3x + 2 = 0 \ (mod\ 10)$. (5 marks)

b) Prove that 7 divides $3^{2n+1} + 2^{n+2}$ for all natural numbers n (5 marks)

c) Find x such that $x \equiv 3 \ mod\ 5$ and $x \equiv 7 \ mod\ 11$ (5 marks)

QUESTION FIVE

a) Prove by induction $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ (5 marks)

b) Prove that if a, b, n are positive integers, then the greatest common divisor of a and b is the same as the greatest common divisor of $a + nb$ and b . Use your results to find the greatest common divisor of 3456 and 246. (10 marks)

c) State the Euclidean Algorithm process of finding greatest common divisor and use it to find the greatest common divisor 23 and 29. (5 marks)