(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: MAP 221

COURSE TITLE: LINEAR ALGEBRA II

DATE: 11/05/2022 **TIME: 9:00 AM - 11:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE

a) State prove that $\langle 0,u \rangle = \langle u,0 \rangle = 0$ (4 marks)

b) Let $u = (2,-1,1)$ and $v = (1,1,2)$. Find $\langle u,v \rangle$ and the angle between these vectors. (3 marks)

c) Show that the following set is a basis for IR^4 (6 marks)

$S = \{ (2,3,2,-2), (1,0,0,1), (-1,0,2,1), (-1,2,-1,1) \}$

d) Let $S_1 = \{ u_1 = (1,-2), u_2 = (3,-4) \}$

$S_2 = \{ v_1 = (1,3), v_2 = (3,8) \}$

Find the change of basis matrix Q from S_2 to S_1 (10 marks)

e) Find the eigenvalues of the following matrix

$$\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

(7 marks)

QUESTION TWO

a) Given a vector $\mathbf{v} = (a, b, c,)$ in IR^3

i) Show that $\cos \alpha = \frac{a}{\|\mathbf{v}\|}$ (2 marks)

ii) Find $\cos \beta$ (2 marks)

iii) Find $\cos \gamma$ (2 marks)

iv) Show that $\frac{\mathbf{v}}{\|\mathbf{v}\|} = (\cos \alpha, \cos \beta, \cos \gamma)$ (2 marks)

v) Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ (2 marks)

b) Let V be a vector space, $\mathbf{u} \in V$ and α is a scalar. Prove that the following properties hold.

i) $0\mathbf{u} = 0$ (2 marks)

ii) $\alpha 0 = 0$ (2 marks)

iii) $(-1)\mathbf{u} = -\mathbf{u}$ (2 marks)

iv) If $\alpha\mathbf{u} = 0$ then $\alpha = 0$ or $\mathbf{u} = 0$ (4 marks)

QUESTION THREE

Apply the Gram-Schmidt orthonormalization process to the following basis for IR^3

$B = \{ (1,1,0), (1,2,0), (0,1,2)\}$ (20 Marks)

QUESTION FOUR

a) For the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Verify that $x_1 = (-3,-1,1)$ and $x_2 = (1,0,0)$ are eigen vectors of A and find their corresponding eigenvalues (8 marks)

b) Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Find the eigenvalues of A and their associated eigenvectors (12 marks)

QUESTION FIVE

a) Find the quadratic form of A given that

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (5 marks)

b) Show that $A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is a positive matrix (5 marks)

c) Find the matrix representation of the linear map $T: IR^2 \rightarrow IR^2$ defined by $T(x,y) = (3x-4y, x+5y)$ with respect to the basis

$B = \{ v_1 = (1,3), v_2 = (2,5)\}$ (10 marks)