(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 252**

COURSE TITLE: ENGINEERING MATHEMATICS II

DATE: 11/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30 MARKS)

a) If $\vec{A} = xz^3 \underset{\sim}{i} - 2x^2yz \underset{\sim}{j} + 2yz^4 \underset{\sim}{k}$, find $\nabla \times \vec{A}$ at the point $(1,-1,1)$. (3 marks)

b) A particle moves so that its position vector is given by $\vec{r} = 2\cos\omega t \underset{\sim}{i} + 2\sin\omega t \underset{\sim}{j}$,

where ω is a constant. Show that the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$. (4 marks)

c) If $B = 3z^2 + 4i$, find Laplacian of B (4 marks)

d) If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $(1,-2,-1)$. (3 marks)

e) Show that for the complex variable z the following formula is valid:
$\sin 2y = 2\sin y \cos y$ (3 marks)

f) Evaluate $\int_v \vec{F}\, dv$ where v is the region bounded by the planes:

$x = 0, x = 2, y = 0, y = 3, z = 0, z = 4$ and $\vec{F} = xy \underset{\sim}{i} + z \underset{\sim}{j} - x^2 \underset{\sim}{k}$. (4 marks)

g) Find the work done in moving a body along a straight line from $(5,3,-1)$ to $(3,-2,2)$ in a force field given by $\vec{F} = 2\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$. (3 marks)

h) Find the following: $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \bullet 3\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (3 marks)

i) Prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic (3 marks)

QUESTION TWO (20 MARKS)

a) Find $\nabla\phi$ if $\phi = \frac{1}{r}$. (4 marks)

b) Classify according to type and determine the characteristics of the following p.d.e (4 marks)

c) Calculate e^z when $z = 1 + \frac{\pi}{4}i$ (3 marks)

d) If $\vec{F} = (2xy + z^3)\underset{\sim}{i} + x^2 \underset{\sim}{j} + 3xz^2 \underset{\sim}{k}$

(i) Show that it is a conservative force field (3 marks)

(ii) Find the scalar potential (3 marks)

(iii) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,-1,4)$ (3 marks)

QUESTION THREE (20 MARKS)

a) If $U = a + ib$ and $V = c + id$, prove that $\overline{UV} = \overline{U} \times \overline{V}$ (5 marks)

b) Find the Fourier series expansion for the following periodic function
$f(x) = x^2 : -\pi \leq x \leq \pi$ (8 marks)

c) A particle moves along a curve whose parametric equations are
$x = e^{-t}, y = 3\cos 2t, z = 3\sin 2t$, where t is time. Determine:

(i) Velocity at time t (4 marks)

(ii) Acceleration at time t (3 marks)

QUESTION FOUR (20 MARKS)

a) Show that the function $f(x) = x^2 - y^2 - 2ixy$ is analytic in the entire complex plane (5 marks)

b) Given that: $\phi = 2x^3y^2z^4$, find $\nabla \bullet \nabla\phi$ (5 marks)

c) Find the characteristics of the following equation and reduce it to appropriate

standard form and then obtain the general solution $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Verify Stoke's theorem for $\vec{A} = 2z\underset{\sim}{i} + 3x\underset{\sim}{j} + 5y\underset{\sim}{k}$ and S is upper part of the sphere given by $z = 4 - x^2 - y^2$ (8 marks)

b) A scalar field $v = xyz$ exists over a curved surface defined by $x^2 + y^2 = 4$ between the planes $z = 0$ and $z = 3$ in the first octant. Evaluate $\int_s v d\vec{s}$ over this surface. (4 marks)

c) Find the Fourier series representing $f(x) = x : 0 \leq x \leq 2\pi$ (8 marks)

END