(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

YEAR FOUR SEMESTER SPECIAL/SUPPLIMENTARY EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE

COURSE CODE: STA 442

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 18/01/2022 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer **Questions ONE** and **ANY OTHER TWO**.

QUESTION ONE [30 MARKS]

(a) Explain the following terms

(i)	Principle component analysis	(1mk)
(ii)	Random vector	(1mk)
(iii)	Multivariate data	(1mk)

(b) Given the joint pdf of random variables X, Y and Z as

$$f(x,y,z) = \begin{cases} e^{-x-y-z}\ ; 0 < x < \infty, 0 < y < \infty, 0 < z < \infty \\ 0\ ; elsewhere \end{cases}$$

Find the joint cumulative distribution function (cdf) of the three random variables. (5mks)

(c) Let $\underline{x} = [1, 3, 2]$ and $\underline{y} = [-2,\ 1, -1]$ find

(i)	The length of $\underline{x}$	(1mk)
(ii)	The angle between $\underline{x}$ and $\underline{y}$	(3mks)
(iii)	The length of the projection of $\underline{x}$and $\underline{y}$	(1mk)

(d) Let $A = \begin{bmatrix} 3 & -\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$

(i)	Is A symmetric? Give reason	(1mk)
(ii)	Show that A is positive definite	(4mks)

(e) Consider the following $n = 7$ observations on $p = 2$ variables

x_1	3	4	2	6	8	2	5
x_2	5	5.5	4	7	10	5	7.5

(i) Compute the sample means $\bar{x}_1$ and $\bar{x}_2$ and the sample variances S_{11} and S_{22} (4mks)

(ii) Compute the sample covariance S_{12} and the sample correlation coefficient r_{12} and interpret these quantities (5mks)

(iii) Display the sample mean array $\bar{x}$, the sample correlation array R and the sample variance-covariance S_{12} (3mks)

QUESTION TWO (20MARKS)

(a) Let $\underline{x}$ be a p-variate random vector with mean vector $\underline{\mu}$ and variance covariance matrix Σ, show that $E(\underline{XX'}) = \Sigma + \underline{\mu\mu}'$, hence show that $E(\underline{X}'A\underline{X}) = trace(A\Sigma) + \underline{\mu}'A\underline{\mu}$ where A is a symmetric matrix of constants. (8mks)

(b) Find the symmetric matrix A for a quadratic form $Q(X_1, X_2, X_3) = 9X_1^2 + 16X_1X_2 + X_2^2 + 8X_1X_3 + 6X_2X_3 + 3X_3^2$. Hence obtain the expected value of $Q(X_1, X_2, X_3)$ and $E(X'AX)$ given that $\boldsymbol{\mu} = \begin{pmatrix} 6 \\ 7 \\ 3 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 4 & 3 \\ 5 & 3 & 9 \end{pmatrix}$ (12mks)

QUESTION THREE (20 MARKS)

(a) Assume $\underline{x}' = (x_1, x_2, x_3)$ is normally distributed with mean vector $\underline{\mu} = (1, -1, 2)$ and variance matrix $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. Find the distribution of $3x_1 - 2x_2 + x_3$ (6mks)

(b) Show that the sample mean is an unbiased estimator of $\underline{\mu}$ and that the sample variance is biased estimator of matrix Σ (9mks)

(c) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$$

Obtain

(i) Square root of $\Sigma = \left(V^{\frac{1}{2}}\right)$ (2mk)

(ii) Inverse of the square root $\Sigma = \left(V^{\frac{1}{2}}\right)^{-1}$ (1mk)

(iii) Correlation matrix ρ defined by

$$\rho = \left(V^{\frac{1}{2}}\right)^{-1} \Sigma \left(V^{\frac{1}{2}}\right)^{-1}$$ (2mks)

QUESTION FOUR (20 MARKS)

(a) Find the maximum likelihood estimators of the mean vector $\underline{\mu}$ and covariance matrix Σ based on the data matrix (6mks)

$$x = \begin{bmatrix} 4 & 1 \\ -1 & 3 \\ 3 & 5 \end{bmatrix}$$

(b) Given the data matrix $x = \begin{bmatrix} 1 & 9 & 10 \\ 4 & 12 & 16 \\ 2 & 10 & 12 \\ 5 & 8 & 13 \\ 3 & 11 & 14 \end{bmatrix}$

Define $X_c = X - 1\,\bar{x}'$ as the mean corrected data matrix.

(i) Obtain the mean corrected data matrix (4mks)

(ii) Obtain the sample covariance matrix (4mks)

(iii) The generalized variance and hence verify that columns of mean corrected data matrix are linearly dependent. (3mks)

(iv) Specify a vector $a' = [a_1\ a_2 a_3]$ that establishes the linear dependence (3mks)

QUESTION FIVE (20 MARKS)

(a) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

(i) Obtain the population correlation matrix (ρ) and $V^{\frac{1}{2}}$ (6mks)

(ii) Multiply your matrices to check the relation $V^{\frac{1}{2}}\rho V^{\frac{1}{2}}$ (4mks)

(b) Let $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

(iii) Is A symmetric? Give reason. (1mk)

(iv) Obtain Eigen value (3mks)

(v) Show that A is positive definite (6mks)

END