(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 448

COURSE TITLE: STOCHASTIC PROCESSES II

DATE: 17/01/2022 **TIME:** 2:00 - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. *Please Turn Over.*

QUESTION 1: (30 Marks) (COMPULSORY)

a) Let X have the distribution of the geometric distribution of the form

$$Prob\ (X = k) = p_k = q^{k-1}\, p\ , \quad k = 1, 2, 3, \ldots$$

Obtain the probability generating function and hence find its mean and variance **[9mks]**

b) Given that random variable X have probability density function $pr(X = k) = p_k \quad k = 0, 1, 2, 3, \ldots.$ with probability generating function $P(S) = \sum_{i=1}^{\infty} p_k s^k$ and $q_k = p_k(X = k) = p_{k+1} + p_{k+2} + p_{k+3} + \cdots$ with generating function $\phi(s) = \sum_{i=1}^{\infty} q_k s^k$

Show that $(1 - s)\phi(s) = 1 - p(s)$ and that $E(X) = \phi(1)$ **[6mks]**

c) Find the generating function for the sequence $\{0, \ 0, 0, 5, 5, 5, 5, \ldots\}$ **[2mks]**

d) Define the following terms

i. Absorbing state **[1mk]**

ii. Irreducible markov chains **[1mk]**

iii. Period of a state of markov chains **[1mk]**

e) Classify the state of the following stochastic markov chain

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

[10mks]

QUESTION 2: (20 Marks)

The difference – differential equation for pure birth process are

$P'_n(t) = \lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t), \quad n \geq 1$ and

$P'_0(t) = -\lambda_0 p_0(t), \ n = 0.$

Obtain $P_n(t)$ for a non – stationary pure birth process (Poisson process) with $\lambda_n = \lambda$ given that

$$P_0(t) = \begin{cases} 1 & for\ n = 0 \\ 0 & otherwise \end{cases}$$

Hence obtain its mean and variance

QUESTION 3: (20 Marks)

a) Let X have a Poisson distribution with parameter λ i.e.

$$Prob\ (X = k) = p_k = \frac{e^{-\lambda}\lambda^k}{k!},\quad k = 0, 1, 2, 3, \ldots$$

Obtain the probability generating function of X and hence obtain its mean and variance **[5mks]**

b) Using Feller's method, find the mean and variance of the difference – differential equation

$P'_n(t) = -n(\lambda + \mu)p_n(t) + (n-1)\lambda p_{n-1}(t) + \mu(n+1)p_{n+1}(t),\ n \geq 1$ given $m_1(t) = \sum_{n=0}^{\infty} n p_n(t)$, $m_2(t) = \sum_{n=0}^{\infty} n^2 p_n(t)$ and $m_3(t) = \sum_{n=0}^{\infty} n^3 p_n(t)$ conditioned on $p_1(0) = 0,\ p_n(0) = 0,\ n \neq 0$ **[14mks]**

QUESTION 4: (20 Marks)

a) Define the following terms

i. Transient state **[1mk]**

ii. Ergodic state **[1mk]**

iii. Recurrent state **[1mk]**

b) Classify the state of the following transitional matrix of the markov chains

$$\begin{array}{c} \\ E_1 \\ E_1 \\ E_1 \\ \vdots \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccccc} E_1 & E_2 & E_3 & E_4 & E_5 & \cdots \end{array} \\
\begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & \cdots \\ 1/2 & 0 & 1/2 & 0 & 0 & \cdots \\ 1/2 & 0 & 0 & 1/2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/2 & 0 & 0 & 0 & 0 & \cdots \end{bmatrix} \end{array}$$

[17mks]

QUESTION 5: (20 Marks)

a) Let $\boldsymbol{X}$ have a binomial distribution with parameter $\boldsymbol{n}$ and $\boldsymbol{p}$ i.e.

$$\boldsymbol{Prob}\ (\boldsymbol{X} = \boldsymbol{k}) = \boldsymbol{p_k} = \binom{n}{k}\ p^k\ q^{n-k}\ , \qquad k = 0, 1, 2, 3, \ldots, n$$

Obtain the probability generating function of $\boldsymbol{X}$ and hence find its mean and variance. **[7mks]**

b) Consider a series of Bernoulli trials with probability of success $\boldsymbol{P}$. Suppose that $\boldsymbol{X}$ denote the number of failures preceding the first success and $\boldsymbol{Y}$ the number of failures following the first success and preceding the second success. The joint pdf of $\boldsymbol{X}$ and $\boldsymbol{Y}$ is given by

$$\boldsymbol{P_{ij} = pr\{x = j, y = k\} = q^{j+k}p^2 \quad j, k = 0, 1, 2, 3, \ldots}$$

i. Obtain the Bivariate probability generating function of $\boldsymbol{X}$ and $\boldsymbol{Y}$ **[3mks]**

ii. Obtain the marginal probability generating function of $\boldsymbol{X}$ **[2mks]**

iii. Obtain the mean and variance of $\boldsymbol{X}$ **[4mks]**

iv. Obtain the mean and variance of $\boldsymbol{Y}$ **[4mks]**