

120



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAT 321/MAA 311

COURSE TITLE: ODE I

DATE: 18/05/2022

TIME: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Using examples define the following terms
- i) A differential equation (2 mks)
 - ii) The order of a differential equation (2 mks)
 - iii) The degree of a differential equation (2 mks)
- b) Solve the linear fractional differential equation
 $(2y - 3x - 7)dx + (2x + 3y + 9)dy = 0$ (6 mks)
- c) Solve the non-homogeneous differential equation
 $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 4\cos x$ (6 mks)
- d) Test for exactness and solve the differential equation
 $(xe^{xy} - 6xy^2 - 2y)dy + (ye^{xy} - 2y^3)dx = 0 \quad y(0) = 2$ (6 mks)
- e) A loaf of bread at $100^{\circ}C$ is placed in a room of $40^{\circ}C$ constant temperature. What should be the constant of proportionality in order that the bread be at $60^{\circ}C$ after 10 minutes? (6 mks)

QUESTION TWO [20 MARKS]

- (a) Using the integrating factor solve the differential equation
 $(x^2y - x)dy + (y + 3x^2)dx = 0$ (9 mks)
- (b) Solve the following Bernoulli's equation
 $x\frac{dy}{dx} - (1 + x)y = xy^2 \quad y(0) = 2$ (7 mks)
- (c) The sum of Kshs 3600 is invested at a rate of 13% per annum compounded continuously. What will be the amount after 10 years? (4 mks)

QUESTION THREE [20 MARKS]

(a) Obtain the differential equation having a solution as

$$y = C_1 e^{3x} - C_2 e^{-4x} + C_3 \quad (7 \text{ mks})$$

(b) Solve the differential equations using appropriate method

(i) $e^x y \frac{dy}{dx} + x + xy^2 = 0$ (5 mks)

(ii) $y'' + 2xy' = 2xe^{-x^2}$ (8 mks)

QUESTION FOUR [20 MARKS]

a) Show that it is homogeneous and solve the differential equation

$$(y^2 + 2xy)dx - (x^2 + y^2 + xy)dy = 0 \quad (6 \text{ mks})$$

b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 4x^2 + 6e^x \quad (7 \text{ mks})$$

c) Solve the equation by method of variation of parameter

$$y'' + 2y' + y = 3e^{-x} \quad (7 \text{ mks})$$

QUESTION FIVE [20 MARKS]

(a) Verify that the functions x and xe^x are linearly independent solutions of the

homogeneous part of the differential equation $x^2 \frac{d^2y}{dx^2} - x(2+x) \frac{dy}{dx} + (x+2)y = 2x^3$
(7 mks)

(b) Perform the multiplication $(D+2)(D^2-2D+5)$ (6 mks)

(c) Use the method of undetermined coefficients to solve

$$y'' - 3y' - 4y = 3x + 2 \quad (7 \text{ mks})$$