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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
MATHEMATICS

COURSE CODE: MAP 411

COURSE TITLE: TOPOLOGY

DATE: 16/05/2022

TIME: 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a. Define the following
- i. Cauchy sequence (2marks)
 - ii. Complete metric space (1mark)
 - iii. Closed subset (2marks)
- b. What are the basic properties of open balls (5 marks)
- c. Show that in a metric space X , a subset $Z \subset X$ is closed if and only if for every sequence $p_1, p_2, \dots \in Z$ that converges to a point $p \in X$, we have $p \in Z$. (8marks)
- d. Show that a function $f: X \rightarrow Y$ is continuous if and only if for all open sets $U \subset X$, the preimage $f^{-1}(U) \subset X$ is open (12marks)

QUESTION TWO (20 MARKS)

- a. Define the following
- i. Topological space (1marks)
 - ii. Discrete topology (1marks)
 - iii. Finer topology (2marks)
 - iv. Basis (3marks)
- b. What are the basic properties of topology (3 marks)
- c. Show that the topology generated by a basis B is indeed a topology (10marks)

QUESTION THREE (20 MARKS)

- a. Define the following
- i. Subspace topology (2mark)
 - ii. Product topology (2marks)
 - iii. Order topology (4marks)
 - iv. Bounded metric space (1marks)
- b. Show that T_A is indeed a topology on A . Further more if B is a basis for T_X then $\{B \cap A : B \in B\}$ is a basis for T_A (6marks)
- c. Let B be a basis. Show that open sets of T are all unions of sets in B (5marks)

QUESTION FOUR (20 MARKS)

- a. Define the following
- i. Interior of a subset (2marks)
 - ii. Boundary of a subset (2marks)
- b. Show that a function $f: X \rightarrow Y$ is continuous if and only if for all basis element $B \subset Y$ for the topology on Y , $f^{-1}(B) \subset X$ is open (4 marks)
- c. Show that a function $f: X \rightarrow Y$ is continuous if for every point in X , there is an open set of X on which f is a function (6marks)
- d. Show that $\bar{A} = A \cup \{\text{limit points of } A\}$ (6marks)

QUESTION FIVE (20 MARKS)

- a. Define the following
- i. Hausdorff (2marks) topology
 - ii. Homeomorphism (2marks)
 - iii. Dense subset (1marks)
 - iv. Embedding (3 marks)
- b. Show that if X is Hausdorff, then every sequence converges to at most one limit. (6marks)
- c. Show that a map $f: Z \rightarrow \prod X_i$ is continuous if the component $f_i: Z \rightarrow X_i$ is continuous for all i . (6marks)