(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 446**

COURSE TITLE: **BAYESIAN STATISTICS**

DATE: 4/10/2021 **TIME:** 9:00 - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

1. (a) State and explain any two differences between frequentist and Bayesian statistics (4 mks)

   (b) Define the following terms: Informative prior, hyper-parameters, Conjugate prior (3 mks)

   (c) A bag contains two unbiased coins and a biased coin with probability for heads $P(H) = 0.3$. A coin is chosen at random from the bag and tossed three times. If two heads and a tail are obtained, what is the probability of the event B that the coin is biased. (3 mks)

   (d) Describe the procedure for testing hypotheses in Bayesian inference (5 mks)

   (e) Let $X \sim B(n, p)$. Assume the prior distribution of p is uniform on [1,0]. Show that the posterior is essentially the likelihood function. (5 mks)

   (f) Suppose $X_1, ..., X_n$ is a random sample from geometric distribution with parameter p, $0 \leq p \leq 1$. Assume that the prior distribution of p is a beta distribution with $\alpha = 4$ and $\beta = 4$. Find

      i. the posterior distribution of p (2 mks)

      ii. the Bayes estimate under quadratic loss function (3 mks)

   (g) Let $X_1, ..., X_n$ be normally distributed random sample with mean μ and variance σ^2) with prior $\pi(\mu)$ having $N(\mu_0, \sigma_0^2)$ distribution with known σ^2. Obtain the posterior distribution of μ. (5 mks)

QUESTION TWO (20 MARKS)

2. (a) Suppose $X_1, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$ with $\sigma^2 = 4$. If the prior pdf of μ is $\pi(\mu) \sim N(0, 1)$. Find 95 percent credible interval of μ. (10 mks)

   (b) Let $X \sim N(\mu, \sigma^2)$, where σ^2 is known and μ is unknown. If μ behaves as a random variable whose probability distribution is $\pi(\mu)$ and also normally distributed with mean μ_p and variance σ_p^2, both assumed to be known. Find the mean and variance of the posterior density, $f(\mu|x)$ (10 mks)

QUESTION THREE (20 MARKS)

3. (a) The cross-fertilized plant produce taller offspring than the self-fertilized plants. In order to obtain an estimate on the proportion θ of cross-fertilized that are taller, a statistical researcher observes a random sample of 15 pairs of plants that are exactly the same age. Each pair is grown in the same conditions with some cross-fertilized and others are self-fertilized. Based on previous experience, the researcher believes that the following are possible values of θ and that the prior probability for each value of θ is $p(\theta)$

θ	0.70	0.72	0.74	0.76	0.78	0.80
$p(\theta)$	0.12	0.16	0.23	0.24	0.15	0.10

   i. Find the Bayesian estimate for the uniform prior (8 mks)
   ii. Find the Bayesian estimate for the informative prior (4 mks)
   iii. Comment on the Bayesian estimate in (i) and (ii) above (2 mks)

   (b) Suppose $X_1, ..., X_n$ is a sample from geometric distribution with parameter p, $0 \leq p \leq 1$. Assume that the prior distribution of p is beta with $\alpha = 4$ and $\beta = 4$. Find

   i. the posterior distribution of p (2 mks)
   ii. the Bayes estimate under quadratic loss function (4 mks)

QUESTION FOUR (20 MARKS)

4. (a) A Bernoulli random variable X with a probability θ for success is known to be either 0.3 or 0.6. It is desired to test the null hypothesis $H_0 : \theta = 0.3$ against the alternative $H_1 : \theta = 0.6$ using the Bayes 0.05 test assuming the vague prior probability distribution for θ: $P(\theta = 0.3) = P(\theta = 0.6) = 0.5$. A sample of 30 trials on X yields 16 successes.

   i. Find the posterior probability of the null hypothesis (5 mks)

   ii. Check the rejection criterion of the Bayes 0.05 test (3 mks)

   (b) In the past a hundred days, the sun has been predicted to rise the next morning or not. Each evening, the sun is said to rise the next morning ($\hat{R}$) and found right (R) all these days. Suppose on the 10^2 evenings it was predicted that the sun will rise on the next day. What is the probability that the sun will rise the next day? (12 mks)

QUESTION FIVE (20 MARKS)

5. (a) A student taking a standardized test is classified as talented if he or she scores at least 80 out of a possible score of 100. Otherwise the student is classified as not talented. Suppose the prior distribution of the scores of all students is normal with mean 70 and standard deviation 10. It is believed that scores will vary each time the student takes the test and that these scores can be modeled as a normal distribution with mean μ and variance σ^2. Suppose the student takes the test and scores 85. Test the hypothesis that the student can be classified as a talented student. (10 mks)

   (b) Let $P(\theta = 1) = P(\theta = 0) = \frac{1}{2}$. That is, $P(goodweather) = P(badweather) = \frac{1}{2}$. Suppose the following record on the meteorologists predictions is available. The meteorologist predicts good weather ($\hat{G}$), given the weather is good, $\frac{2}{3}$ of the time, that is, $P(\hat{G}|G) = \frac{2}{3}$, and predicts bad weather, given the weather is bad, $\frac{3}{4}$ of the time, that is, $P(\hat{B}|B) = \frac{3}{4}$.

   i. Given that the meteorologist predicts good weather, what is the probability that the weather will turn out to be good? (5 mks)

   ii. Given that the meteorologist predicts bad weather, what is the probability that the weather will turn out to be bad? (5 mks)