KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: SPC 323

COURSE TITLE: MATHEMATICAL PHYSICS II

DURATION: 2 HOURS

DATE: 7/10/2021 **TIME:** 2:00-4:00 PM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other **TWO** (2) Questions.
- Question **ONE** carries **30 MARKS** and the remaining carry **20 MARKS** each.
- ALL Symbols have their usual meaning

QUESTION ONE (30MARKS)

a) Use complex analysis to evaluate
   i) $(1+i)^8$ (5marks)
   ii) $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ (5marks)

b) i) Show that a complex function $f(z)$ is analytic at z_o in R (3marks)
   ii) Show that $f(z) = z^*$ is continuous at z_o but $\frac{dz^*}{dz}$ does not exist (4marks)

c) Show that if $f(z) = u(x,y) + i(x,y)$ the complex line integral can be expressed as a real integral (4marks)

d) Show that the Cauchy-Riemann conditions hold for $f(z) = z^2$at all points. (4marks)

e) Define the gamma function $\sqrt{(p)}$ (1mark)

f) Determine if the following series converge $\sum_{n=0}^{\infty}(\log_{\pi} 2)^n$ (4marks)

QUESTION TWO (20MARKS)

a) Write down the relationship between the Beta and gamma function and use it to evaluate $\int_0^{\infty} \frac{x^3}{(1+x)^5} dx$ (5marks)

b) Write down the Rodrigue's formula and hence generate the first three Legendre polynomials (5marks)

c) Evaluate the integral $\int_C (Z^*)^2 dz$ where C is a straight line joining the points $z = 0$ and $z = 1 + 2i$
(5marks)

a) Use the Laplace transform of the first derivative to show that $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$ (5marks)

QUESTION THREE (20MARKS)

a) Use the Laplace transform tables to work out $\mathcal{L}\{3 \sin^2 x\}$ (6marks)

b) Show that the complex sequence whose n^{th} term is $Z_n = \frac{n^2-2n+3}{3n^2-4} + i\frac{2n-1}{2n+1}$ converges to $\frac{1}{3} + i$ (4marks)

c) Use the calculus of residues to show that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$ (10marks)

QUESTION FOUR (20MARKS)

a) Use the calculus of residues to show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ where $a > b > 0$
(12marks)

b) Evaluate the integral $\int_C \frac{dz}{(z-z_o)^{n+1}}$ where C is a circle of radius r and center at z_o and n is an integer. (8marks)

QUESTION FIVE (20MARKS)

a) Find the circle of converge of

i) $\sum_{n=0}^{\infty} nz^n$ (4marks)

ii) $\sum_{n=0}^{\infty}(z+5i)^{2n}(n+1)^2$ (7marks)

b) Use the Laplace transform of the first derivative to work out $\mathcal{L}\{k\}$ (4marks)

c) Find the poles and the corresponding residue of $f(z)=\frac{e^z}{z^2+a^2}$ (5marks)