(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAA 223**

COURSE TITLE: CLASSICAL MECHANICS

DATE: 5/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms:
   i) Kinetic energy (1 mark)
   ii) Kinematics (1 mark)
   iii) Work (1 mark)
   iv) Statics (1 mark)
   v) Momentum (1 mark)
   vi) Impulse (1 mark)

b) Find the work done in moving a particle along a straight line from $(-4,3,2)$ to $(2,-1,3)$ in a force field given by $\vec{F} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ (4 marks)

c) Find the constant force needed to accelerate a mass of $10kg$ moving along a straight line from a speed of $54km/h$ to a speed of $108km/h$ in 5 seconds. (4 marks)

d) An electric motor rated $3kW$ is used to lift bales of hay to a store in a dairy farm. A single bale has a mass of $5kg$. If the store is $5m$ above the ground, how many bales can the motor raise in 3 minutes? (4 marks)

e) The three motion laws of Newton are the actions of mechanics. (3 marks)
   (A) State these laws
   (B) A particle of mass 2 units moves in a force field $\vec{F}$ depending on time t given by $\vec{F} = 24t^2\,\underset{\sim}{i} + (36t - 16)\,\underset{\sim}{j} - 12t\,\underset{\sim}{k}$. Assuming that at $t = 0$, the particle is located at $\vec{R}_0 = 3\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ and has a velocity $\vec{V}_0 = 6\underset{\sim}{i} + 16\underset{\sim}{j} - 8\underset{\sim}{k}$, find
      (i) Velocity at any time t (5 marks)
      (ii) Position at any time t (4 marks)

QUESTION TWO (20 MARKS)

a) A particle of mass m moves in the $xy-plane$ so that its position vector is given by $\underset{\sim}{r} = a\cos\omega t\,\underset{\sim}{i} + b\sin\omega t\,\underset{\sim}{j}$ where a, b and ω are positive constants and $a > b$
   (A) Show that the:
      (i) particle moves in an ellipse (4 marks)
      (ii) force acting on the particle is always directed towards the origin (3 marks)
   (B) Find the kinetic energy of the particle at points A and B (3 marks)
   (C) Find the work done by the force field in moving the particle from A to B (2 marks)

(D) Show that the total work done in moving the particle around the ellipse is zero (3 marks)

b) What is meant by the terms:
   i) Parking orbit (1 mark)
   ii) Escape velocity (1 mark)

c) The moon revolves about the earth in $30 days$. Assuming that the orbit is circular and has a radius of $4.0\times10^5\ km$, calculate the acceleration of the moon towards the earth. (3 marks)

QUESTION THREE (20 MARKS)

a) A constant force $\vec{F}$ acting on a particle of mass m changes the velocity from v_1 to v_2 in time τ.

   (i) Prove that $\vec{F}=\dfrac{m(v_2-v_1)}{\tau}$ (5 marks)

   (ii) Does the result in (i) hold if the force is variable? (3 marks)

b) As an example of recoil, consider a radioactive decay in which an alpha particle, the nucleus of Helium atom of mass number 4 and atomic number 2, is emitted from a Uranium-238 nucleus originally at rest with a speed of $1.4\times10^7\ m/s$ and kinetic energy of $4.1 MeV$. Find

   (i) the velocity of the residual Thorium-234 nucleus. (4 marks)

   (ii) kinetic energy of Thorium (4 marks)

c) Find the work done in moving a particle along the vector $\vec{S}=5\underset{\sim}{i}+3\underset{\sim}{j}-2\underset{\sim}{k}$ if the force applied is $\vec{F}=3\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Due to a force field a mass of $5 units$ moves along a space curve whose position vector is $\underset{\sim}{r}=\left(2t^3+t\right)\underset{\sim}{i}+\left(3t^4-t^2+8\right)\underset{\sim}{j}-12t^2\underset{\sim}{k}$. Find

   (i) Velocity (3 marks)

   (ii) Momentum (3 marks)

   (iii) Acceleration (2 marks)

   (iv) Force at any time t (2 marks)

b) State Kepler's laws of planetary motion (3 marks)

c) Given that the mass of the earth is $6\times10^{24}\ kg$ and the gravitational constant is $6.7\times10^{-11}\ m^3kg^{-1}s^{-2}$. The radius of the earth is $6.4\times10^6\ m$. Calculate the gravitational force on a mass of $5kg$ on the earth's surface. (3 marks)

d) An artificial satellite is rotating about the earth at a height h of $200km$ above the surface of the earth where acceleration due to the gravity $g=9.8ms^{-2}$. The radius of the earth r is $6400km$. Calculate the speed of the satellite. (4 marks)

QUESTION FIVE (20 MARKS)

a) Consider two blocks A and B of mass m_A and m_B connected by a light spring on a frictionless surface. If the masses are pulled apart so that the spring is stretched after which they are released, show that:

(i) The velocity of A is given by $\vec{V}_A = -\vec{V}_B\left(\frac{m_B}{m_A}\right)$ (3 marks)

(ii) The ratio of their kinetic energies is given by $\frac{K_A}{K_B} = \frac{m_B}{m_A}$ (4 marks)

b) Find the work done in moving a particle along the vector $\vec{S} = 5\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$

if the applied force is $\vec{F} = 4\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$ (4 marks)

c) State Newton's law of universal gravitation (1 mark)

d) Given that the acceleration due to gravity on the surface of the moon is $1.7ms^{-2}$ and the radius of the moon is $1.70 \times 10^6 m$. Calculate the mass of the moon. (4 marks)

e) A trolley X of mass $80g$ travelling at $3ms^{-1}$ collides with another trolley Y of twice its mass moving in the opposite direction with a velocity of $0.5ms^{-1}$. If the trolleys stuck together on collision, calculate the common velocity with which they move. (4 marks)

END