(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 424**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATIONS III**

DATE: 4/10/2021 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS

Answer **Questions ONE a**nd Any other **TWO**

QUESTION ONE [30MKS]

a. Define the maximal interval of existence. (3mks)

b. Find the fundamental matrix for the given system of equation, (5mks)

$$x' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x$$

c. Solve the initial value problem and give the interval of existence for the solution (4mks)

$$\frac{dy}{dt} = \frac{4}{(x-1)^{2/3}}, \quad y(0) = -10$$

d. Consider, $\dot{x} = x^2$, $x(0) = 1$. Show that the function $f(x) = x^2 \in C^1(\square)$ and the initial value problem has a unique solution. (5mks)

e. Construct a Lyapunov function of the form $Ax^2 + By^2$ (A&B are constants) for the plane autonomous system (6mks)

$$\frac{dx}{dt} = -x + y^2, \qquad \frac{dy}{dt} = -y + x^2$$

f. Show that the origin is an asymptotically stable node of the linear system (5mks)

$$\frac{dx}{dt} = -5x + y, \qquad \frac{dy}{dt} = x - 5y$$

g. State the Poincare-Bendixon theorem (2mks)

QUESTION TWO [20MKS]

a. Find the y_1, y_2, y_3 such that (7mks)

$$y_1' = y_1 - 3y_2 + 7y_3$$
$$y_2' = -y_1 - y_2 + y_3$$
$$y_3' = -y_1 + y_2 - 3y_3$$

b. Use the fundamental matrix to solve the initial value problem (7mks)

$$x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

c. State and prove the Gronwall's inequality (6mks)

QUESTION THREE [20MKS]

a. Define asymptotic stability (2mks)

b. Define a limit cycle (1mk)

c. Determine the nature of limit cycle for the following system of equations (7mks)

$$x' = \mu x + y + x(x^2 + y^2)$$
$$y' = -x + \mu y + y(x^2 + y^2)$$

d. Verify the Taylor's series for sin t and cos t by applying the Picard iteration to the first order system corresponding to the second order initial value problem (10mks)

$$x'' = x, \quad x(0) = 0, \quad x'(0) = 1$$

QUESTION FOUR [20MKS]

a. For any continuously differentiable function $V = V(x, y)$, each integral of the system

$$\frac{dx}{dt} = \frac{\partial V(x, y)}{\partial y}, \qquad \frac{dy}{dt} = -\frac{\partial V(x, y)}{\partial x}$$

Lies on some level curve $V(x, y) =$ constant. Prove. (5mks)

b. Determine the stability of the zero solution of the system using a method of Lyapunov functions $\frac{dx}{dt} = y - 2x, \quad \frac{dy}{dt} = 2x - y - x^3.$ (5mks)

c. Determine the values of the parameters a, b for which the zero solution of the system is asymptotically stable (10mks)

$$\frac{dx}{dt} = ax + y, \quad \frac{dy}{dt} = x + by.$$

QUESTION FIVE [20MKS]

a. If the vector field X(x, t) satisfies the Lipschitz condition in domain R, then there is at most one solution x(t) of the differential system $x' = X(x, t)$ that satisfies the a given initial condition x(a)=c in R. prove (7mks)

b. Determine the limit cycle of the system (6mks)

$$x' = -y + f(r)x$$
$$y' = x + f(r)y$$

c. An RL circuit has an emf of 5 V, a resistance of 50 Ω, an inductance of 1 H, and no initial current. Find the current in the circuit at any time t. Distinguish between the transient and steady-state current and sketch the results. (7mks)