(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 434**

COURSE TITLE: **DIFFERENTIAL GEOMETRY**

DATE: 1/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30marks)

a). Define the following terms

i). Osculating plane (1 mk)

ii). Regular of a curve (1 mk)

iii). Plane product (1 mk)

iv). Torsion to a curve (1 mk)

b). State and derive the second fundamental form of a surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ whose class is more or equal to 2. (6 mks)

c). Find the unit normal vector to the surface $X(u, \theta) = \langle u\cos\theta, u\sin\theta, 2\theta\rangle$. (6 mks)

d). Find the volume of the parallelepiped spanned by vectors $\boldsymbol{u} = (3,3,7), \boldsymbol{v} = (2,1,-1)$ and $\boldsymbol{w} = (4,2,-3)$ respectively. (4 mks)

e). Determine the first and the second curvature of the curve $\boldsymbol{r}(t) = 2t\boldsymbol{i} + 4\sin t\,\boldsymbol{j} + 4\cos t\,\boldsymbol{k}$. (10 mks)

QUESTION TWO (20marks)

a). Let γ be a curve lying on the surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ where $u = u(t), v = v(t), a \leq t \leq b$. Prove that the length of the arc on the curve is given by $\int_a^b \sqrt{I}\ dt$ where I is the first fundamental form of a surface. (6 mks)

b). Find the torsion to the circular helix $\boldsymbol{r}(t) = \langle 4\cos t, 4\sin t, 2t\rangle$ at $t = \frac{\pi}{2}$. (14 mks)

QUESTION THREE (20marks)

a). Determine the lines of curvature to the helicoid $r(s, t) = \langle s\cos t, s\sin t, bt\rangle$. (12 mks)

b). Determine the arc length of the curve $\boldsymbol{X}(t) = \langle e^{2t}\cos t, e^{2t}\sin t, e^{2t}\rangle$ for $0 \leq t \leq \frac{\pi}{2}$. (5 mks)

c). Given the equation of the surface $X(u, v) = \langle 2u, 2u, uv\rangle$ find its first fundamental form. (3 mks)

QUESTION FOUR (20marks)

a). State and proof the Frenet – Serret formulas to the curve $X = X(s)$. (10 mks)

b). Find the equation of the tangent line and normal plane to the curve $\boldsymbol{X}(t) = (1+t)\hat{e}_1 - t^2\hat{e}_2 + (1+t^3)\hat{e}_3$ at $t = 1$. (5 mks)

c). Find the equation of the rectifying plane $\boldsymbol{X}(t) = \langle t^2, t^2, (1+4t)\rangle$ at $t = 1$. (5 mks)

QUESTION FIVE (20marks)

a). Consider a parametrized surface $X(u,v) = \langle \cos u \sin v, \sin u \sin v, \cos v\rangle$ for $(u,v) \in [0,2\pi) \times [0,\pi]$. Determine the length of the curve $\big(u(t), v(t)\big) = \left(t, \frac{\pi}{2}\right)$ for $0 \le t \le 2\pi$ lying on the surface $X(u,v)$. (6 mks)

b). Find the unit binomial vector to the curve $\boldsymbol{X}(t) = \langle 2t + 2t^3, 3t + \frac{t^2}{2}, 4t^2\rangle$ at $t = 1$. (7 mks)

c). Let $X = X(u,v)$ be surface with directions given in parametric form as $(du{:}\, dv)$ and $(\delta u, \delta v)$ whose tangential vectors are $dX = X_u du + X_v dv$ and $\delta X = X_u \delta u + X_v \delta v$ respectively. Prove that the angle between the two directions is given by

$$\theta = \cos^{-1}\left(\frac{I(d,\delta)}{\sqrt{I(d)}\sqrt{I(\delta)}}\right)$$

Where $I(d,\delta)$ is the first fundamental form of a surface. (7 mks)