(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: MAT 304/MAA 321

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 06/10/21 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i) A complex number (2 mks)
   (ii) A simple closed curve (2 mks)
   (iii) A pole (2 mks)

b) Show that $\lim_{z \to 2} \frac{zi}{3} = \frac{2}{3}i$ (3 mks)

c) Using De Moivre's theorem show that
   $sin4\theta = 4(cos^3\theta sin\theta - sin^3\theta cos\theta)$ (4 mks)

d) Find the sixth roots of $-27i$ and locate them on the complex plane (5 maks)

e) Find the residuals of $f(z) = \frac{z}{(z-1)^3(z+3)}$ at all its poles and hence evaluate
   $\oint_C f(z)dz$ (6 mks)

f) Using the Milne-Thomson method find the real part of the analytic function so that $f(z) = U(x,y) + iV(x,y)$, given that $V(x,y) = x^3 + y^3 + 2x - 3xy^2 - 3x^2y$ (6 mks)

QUESTION TWO (20 MARKS)

a) Given that $w = f(z) = z(z-2)$ find the values of w corresponding to
   $z = 3 - i$ (4 mks)

b) (i) State the Taylor's series of expansion (2 mks)
   (ii) Find the first three terms of the Tailor series expansion for the function
   $f(z) = \frac{z+3}{(z-1)(z-4)}$ about the point $z = 2$ (6 mks)

c) Evaluate $\oint_C \frac{z+5}{z^3-9z} dz$ where C is the circle
   (i) $|z-3| = 2$ (4 mks)
   (ii) $|z+3| = 2$ (4 mks)

QUESTION THREE (20 MARKS)

a) Prove that $\cos(x_1 + x_2) = cosx_1cosx_2 - sinx_1sinx_2$ (5 mks)

b) Given the function $f(z) = 2z^2 + 5z^3 - z + i$
   (i) Express it in the form $f(z) = U(x,y) + iV(x,y)$ (5 mks)
   (ii) Verify the Cauchy Riemann conditions for (i) above (5 mks)

c) Evaluate $\int_C \bar{z}dz$ from $z = 0$ to $z = 3 + 9i$ along the curve C given by $z = t + it^2$ (5 mks)

QUESTION FOUR (20 MARKS)

a) Find the analytic function $w = f(z)$ if its real part is $U(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ and if $f(-1) = -i + 2$ (10 mks)

b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for;

(i) $1 < |z| < 3$ (6mks)

(ii) $|z| > 3$ (4mks)

QUESTION FIVE (20 MARKS)

a) Show that the function $V(x, y) = xe^x cosy - ye^x siny$ is harmonic (6 mks)

b) Prove that $\oint 3zdz = 0$ (7 mks)

c) Verify Green's theorem in the plane for $\oint_C (y^2 - xy^3)dx + (x^2 - 3xy)\, dy$ Where C is a square whose vertices are $(0,0)$, $B(4,0)$, $C(4,4)$ and $D(0,4)$ (7 mks)