(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAA 326**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION II**

DATE: 6/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following. (3 Marks)

i) Bifurcation

ii) Stability

iii) Equilibrium point

b) Show that there exist a unique solution to the differential equation $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$, hence find the unique solution. (5 marks)

c) Find the power series solution of $y^{II} + 4y = 0$ near an ordinary point $x = 0$. (8 marks)

d) Solve the following system of differential equations.

$$X^I = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ (7 marks)

e) Determine the nature of the critical point (0,0) of the linear system

$$\frac{dx}{dt} = 2x - 7y$$

$$\frac{dy}{dt} = 3x - 8y$$

and finally whether or not the point is stable. (4 marks)

f) Compute the Jacobian of the function

$$f(x) = \begin{pmatrix} x_1 + x_1 x_2^2 \\ -x_2 + x_2^2 + x_1^2 \end{pmatrix}$$ (3 marks)

QUESTION TWO (20 MARKS)

a) Use reduction of order method to solve the differential equation by

$$x^2y^{II} + 2xy^I - 2y = 0$$

given that $y = x$ is a solution to the differential equation. (10 Marks)

b) Solve the differential equation defined by

$(x^2 - 1)y^{II} - 2xy^I + 2y = o$given that $y = x$ is a solution of the differential equation. (10 marks)

QUESTION THREE (20 MARKS)

Find the power series solution for the initial value problem

$$xy^{II} + y^{I} + 2y = 0$$

$$y(1) = 2 \text{ and } y^{I}(1) = 2$$

at the ordinary point $x = 1$. (20 Marks)

QUESTION FOUR (20 MARK

a) State the condition for the following critical points to occur and in each case draw the phase portrait
   i) Center. (2 marks)
   ii) Spiral point. (2 marks)

b) Consider a nonlinear system

$$f(x) = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

Analyze the system by

i) Finding the critical points. (4 marks)

ii) Linearize the system and determine the type of critical point it has. Draw the phase portrait in each case. (12 marks)

QUESTION FIVE (20 MARKS)

a) Use matrix method to solve the non-homogenous system of equation. (12 marks)

$$\frac{dx_1}{dt} = 4x_2 - 3x_1 + 2$$

$$\frac{dx_2}{dt} = 2x_1 - x_2 + 2$$

b) Use Elimination method to solve the system. (8 Marks)

$$\frac{dx}{dt} + \frac{dy}{dt} - x = 2t + 1$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + x = t$$